5-6 The Quadratic Formula

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

Write each function in standard form.

1. $f(x) = (x - 4)^2 + 3$

$x^2 - 8x + 16 + 3$

$f(x) = x^2 - 8x + 19$

2. $g(x) = 2(x + 6)^2 - 11$

$2(x^2 + 12x + 36)$

$g(x) = 2x^2 + 24x + 61$

Evaluate $b^2 - 4ac$ for the given values of the valuables.

3. $a = 2$, $b = 7$, $c = 5$

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4. $a = 1$, $b = 3$, $c = -3$

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Objectives

Solve quadratic equations using the Quadratic Formula.

Classify roots using the discriminant.

Vocabulary

discriminant

You have learned several methods for solving quadratic equations: graphing, making tables, factoring, using square roots, and completing the square. Another method is to use the *Quadratic Formula*, which allows you to solve a quadratic equation in standard form.

By completing the square on the standard form of a quadratic equation, you can determine the Quadratic Formula.

Can you complete the square for this equation? →

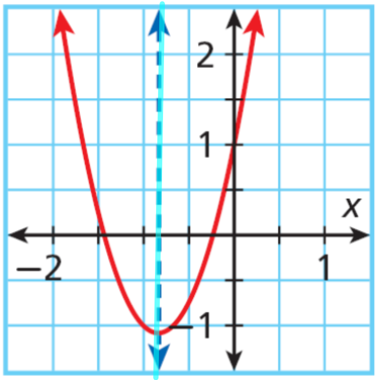
Algebra

	$ax^2 + bx + c = 0 \ (a \neq 0)$
Divide by a.	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
Subtract $\frac{c}{a}$.	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
Complete the square.	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
Factor.	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$
Take square roots.	$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$
Subtract $\frac{b}{2a}$.	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
Simplify.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{b^2}{4a^2} - \frac{c}{a} \quad \frac{4a}{4a}$

$\frac{b^2}{4a^2} - \frac{4ac}{4a^2}$

The symmetry of a quadratic function is evident in the last step, $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. These two zeros are the same distance, $\frac{\sqrt{b^2 - 4ac}}{2a}$, away from the axis of symmetry, $x = -\frac{b}{2a}$, with one zero on either side of the vertex.



The Quadratic Formula

If $ax^2 + bx + c = 0$ $(a \neq 0)$, then the solutions, or roots, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

You can use the Quadratic Formula to solve any quadratic equation that is written in standard form, including equations with real solutions or complex solutions.

Example 1: Quadratic Functions with Real Zeros

$\frac{16}{4} \pm \frac{2\sqrt{10}}{4}$

Find the zeros of $f(x) = 2x^2 - 16x + 27$ using the Quadratic Formula.

$\sqrt{40}$, $\sqrt{4}\sqrt{10}$, $\pm 2\sqrt{10}$, $2\sqrt{10}$

$$2x^2 - 16x + 27 = 0$$ *Set f(x) = 0.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ *Write the Quadratic Formula.*

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(2)(27)}}{2(2)}$$ *Substitute 2 for a, –16 for b, and 27 for c.*

$$x = \frac{16 \pm \sqrt{256 - 216}}{4} = \frac{16 \pm \sqrt{40}}{4}$$ *Simplify.*

$$x = \frac{16 \pm 2\sqrt{10}}{4} = 4 \pm \frac{\sqrt{10}}{2}$$ *Write in simplest form.*

Example 1 Continued

Check Solve by completing the square.

$$2x^2 - 16x + 27 = 0$$

$$2(x^2 - 8x) = -27$$

$$2(x^2 - 8x + 16) = \quad + 2(16)$$

$$2(x-4)^2 = -27 + 32$$

$$2(x-4)^2 = 5$$

$$x = 4 \pm \frac{\sqrt{10}}{2} \checkmark$$

$2(x-4)^2 = 5$

$(x-4)^2 = \frac{5}{2}$

$x - 4 = \pm\sqrt{\frac{5}{2}}$

$x = 4 \pm \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = 4 \pm \frac{\sqrt{10}}{\sqrt{4}}$

Check It Out! Example 1a

Find the zeros of $f(x) = x^2 + 3x - 7$ using the Quadratic Formula.

$x^2 + 3x - 7 = 0$ Set f(x) = 0.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Write the Quadratic Formula.

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-7)}}{2(1)}$$ Substitute 1 for a, 3 for b, and –7 for c.

$$x = \frac{-3 \pm \sqrt{9 + 28}}{2}$$ Simplify.

$$x = \frac{-3 \pm \sqrt{37}}{2}$$ Write in simplest form.

Example 2: Quadratic Functions with Complex Zeros

Find the zeros of $f(x) = 4x^2 + 3x + 2$ using the Quadratic Formula.

$f(x) = 4x^2 + 3x + 2$ Set f(x) = 0.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Write the Quadratic Formula.

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(4)(2)}}{2(4)}$$ Substitute 4 for a, 3 for b, and 2 for c.

$$x = \frac{-3 \pm \sqrt{9 - 32}}{2(4)} = \frac{-3 \pm \sqrt{-23}}{8}$$ Simplify.

$$x = \frac{-3 \pm \sqrt{-23}}{8} = -\frac{3}{8} \pm \frac{\sqrt{23}}{8}i$$ Write in terms of i.

Check It Out! Example 2

Find the zeros of $g(x) = 3x^2 - x + 8$ using the Quadratic Formula.

$3x^2 - x + 8 = 0$ *Set f(x) = 0*

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Write the Quadratic Formula.*

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(8)}}{2(3)}$ *Substitute 3 for a, –1 for b, and 8 for c.*

$x = \frac{1 \pm \sqrt{1-96}}{2(3)} = \frac{1 \pm \sqrt{-95}}{6}$ *Simplify.*

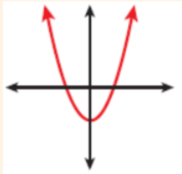
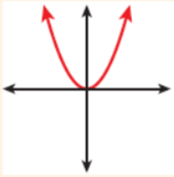
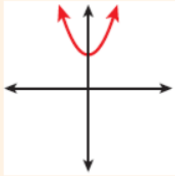
$x = \frac{1 \pm \sqrt{-95}}{6} = \frac{1}{6} \pm \frac{\sqrt{95}}{6}i$ *Write in terms of i.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ← Discriminant

The **discriminant** is part of the Quadratic Formula that you can use to determine the number of real roots of a quadratic equation.

Discriminant

The discriminant of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is $b^2 - 4ac$.

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
two distinct real solutions	one distinct real solution	two distinct nonreal complex solutions
		

Caution!

Make sure the equation is in standard form before you evaluate the discriminant, $b^2 - 4ac$.

Example 3A: Analyzing Quadratic Equations by Using the Discriminant

Find the type and number of solutions for the equation.

$$\boldsymbol{x^2 + 36 = 12x}$$

$$x^2 - 12x + 36 = 0$$

$$b^2 - 4ac$$

$$(-12)^2 - 4(1)(36)$$

$$144 - 144 = 0$$

$$b^2 - 4ac = 0$$

The equation has one distinct real solution.

Example 3B: Analyzing Quadratic Equations by Using the Discriminant

Find the type and number of solutions for the equation.

$$\mathbf{x^2 + 40 = 12x}$$

$$1x^2 - 12x + 40 = 0$$

$$b^2 - 4ac$$

$$(-12)^2 - 4(1)(40)$$

$$144 - 160 = -16$$

$$b^2 - 4ac < 0$$

The equation has two distinct nonreal complex solutions.

Example 3C: Analyzing Quadratic Equations by Using the Discriminant

Find the type and number of solutions for the equation.

$$\mathbf{x^2 + 30 = 12x}$$

$$1x^2 - 12x + 30 = 0$$

$$b^2 - 4ac$$

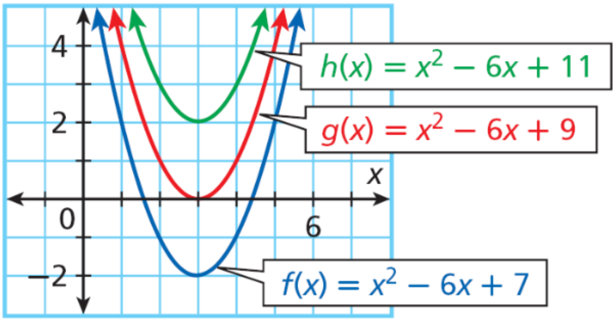
$$(-12)^2 - 4(1)(30)$$

$$144 - 120 = 24$$

$$b^2 - 4ac > 0$$

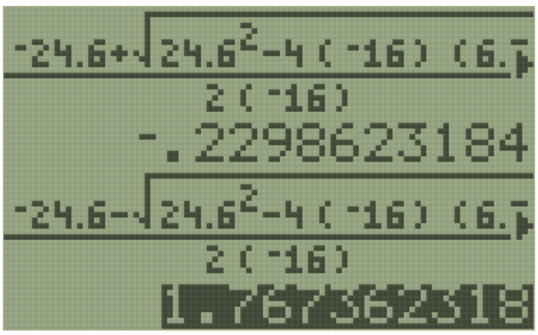
The equation has two distinct real solutions.

The graph shows related functions. Notice that the number of real solutions for the equation can be changed by changing the value of the constant *c*.



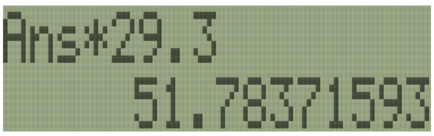
Example 4: Sports Application

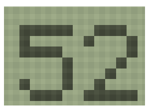
An athlete on a track team throws a shot put. The height *y* of the shot put in feet *t* seconds after it is thrown is modeled by $y = -16t^2 + 24.6t + 6.5$. The horizontal distance *x* in between the athlete and the shot put is modeled by $x = 29.3t$. To the nearest foot, how far does the shot put land from the athlete?



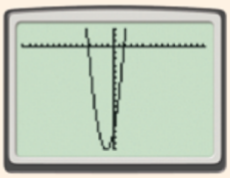
t = 1.77

29.3(1.77)





Properties of Solving Quadratic Equations

Method	When to Use	Examples
Graphing	Only approximate solutions or the number of real solutions is needed.	$2x^2 + 5x - 14 = 0$  $x \approx -4.2$ or $x \approx 1.7$
Factoring	$c = 0$ or the expression is easily factorable.	$x^2 + 4x + 3 = 0$ $(x + 3)(x + 1) = 0$ $x = -3$ or $x = -1$
Square roots	The variable side of the equation is a perfect square.	$(x - 5)^2 = 24$ $\sqrt{(x - 5)^2} = \pm\sqrt{24}$ $x - 5 = \pm 2\sqrt{6}$ $x = 5 \pm 2\sqrt{6}$

Properties of Solving Quadratic Equations

Method	When to Use	Examples
Completing the square	$a = 1$ and b is an even number.	$x^2 + 6x = 10$ $x^2 + 6x + \square = 10 + \square$ $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 10 + \left(\frac{6}{2}\right)^2$ $(x + 3)^2 = 19$ $x = -3 \pm \sqrt{19}$
Quadratic Formula	Numbers are large or complicated, and the expression does not factor easily.	$5x^2 - 7x - 8 = 0$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-8)}}{2(5)}$ $x = \frac{7 \pm \sqrt{209}}{10}$

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Helpful Hint

No matter which method you use to solve a quadratic equation, you should get the same answer.

HW p. 361- 362 (Due Tuesday)

60, 46, 47, 48, 50, 36, 18- 24 even, 29.

HW p. 378 (Due Thursday)

15, 16, 28, 30, 36.

Oct 20-7:58 AM