5-8 Curve Fitting with Quadratic Models

Warm Up
Lesson Presentation
Lesson Quiz

Warm Up

Solve each system of equations.

1. $\begin{cases} 3a + b = -5 \\ 2a - 6b = 30 \end{cases}$

2. $\begin{cases} 9a + 3b = 24 \\ a + b = 6 \end{cases}$

3. $\begin{cases} 4a - 2b = 8 \\ 2a - 5b = 16 \end{cases}$

Objectives

Use quadratic functions to model data.

Use quadratic models to analyze and predict.

Vocabulary

quadratic model

quadratic regression

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Recall that you can use differences to analyze patterns in data. For a set of ordered parts with equally spaced x-values, a quadratic function has constant nonzero second differences, as shown below.

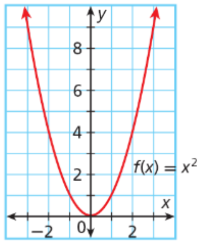
$f(x) = x^2$

Equally spaced x-values

x	−3	−2	−1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9
1st differences	−5	−3	−1	1	3	5	
2nd differences	2	2	2	2	2		

Linear
Quad.

Constant 2nd differences



Example 1A: Identifying Quadratic Data

Determine whether the data set could represent a quadratic function. Explain.

x	1	3	5	7	9
y	–1	1	7	17	31

Find the first and second differences.

Equally spaced x-values

x	1	3	5	7	9
y	–1	1	7	17	31
1st	2	6	10	14	
2nd	4	4	4		

Quadratic function: second differences are constant for equally spaced x-values

Example 1B: Identifying Quadratic Data

Determine whether the data set could represent a quadratic function. Explain.

x	3	4	5	6	7
y	1	3	9	27	81

Find the first and second differences.

Equally spaced x-values

x	3	4	5	6	7
y	1	3	9	27	81

1st 2 6 18 54

2nd 4 12 36

Not a Quadratic function: second differences are not constant for equally spaced x-values

Check It Out! Example 1b

Determine whether the data set could represent a quadratic function. Explain.

x	10	9	8	7	6
y	6	8	10	12	14

Find the first and second differences.

Equally spaced x-values

x	10	9	8	7	6
y	6	8	10	12	14

1st 2 2 2 2

Not a quadratic function: first differences are constant so the function is linear.

Example 2: Writing a Quadratic Function from Data

Write a quadratic function that fits the points (1, –5), (3, 5) and (4, 16).

Use each point to write a system of equations to find *a*, *b*, and *c* in $f(x) = ax^2 + bx + c$.

$A^{-1}B$

$A: \begin{bmatrix} 1 & 1 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{bmatrix}$ $B: \begin{bmatrix} -5 \\ 5 \\ 16 \end{bmatrix}$

(x, y)	$f(x) = ax^2 + bx + c$	System in a, b, c
(1, –5)	$-5 = a(1)^2 + b(1) + c$	$a + b + c = -5$ ❶
(3, 5)	$5 = a(3)^2 + b(3) + c$	$9a + 3b + c = 5$ ❷
(4, 16)	$16 = a(4)^2 + b(4) + c$	$16a + 4b + c = 16$ ❸

Example 2 Continued

Subtract equation ❷ by equation ❶ to get ❹.

❷ $9a + 3b + c = 5$
❶ $-a + -b + -c = +5$
❹ $8a + 2b + 0c = 10$

Subtract equation ❸ by equation ❶ to get ❺.

❸ $16a + 4b + c = 16$
❶ $-a + -b + -c = +5$
❺ $15a + 3b + 0c = 21$

Solve equation ❹ and equation ❺ for *a* and *b* using elimination.

❺ $2(15a + 3b = 21) \rightarrow 30a + 6b = 42$ *Multiply by 2.*
❹ $-3(8a + 2b = 10) \rightarrow -24a - 6b = -30$ *Multiply by −3.*

$6a + 0b = 12$
$a = 2$

Example 2 Continued

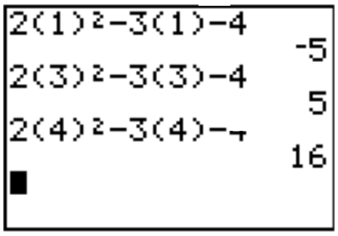
Substitute 2 for *a* into equation ❹ or equation ❺ to get *b*.

$$8(2) + 2b = 10 \qquad 15(2) + 3b = 21$$
$$2b = -6 \qquad 3b = -9$$
$$b = -3 \qquad b = -3$$

Substitute $a = 2$ and $b = -3$ into equation ❶ to solve for *c*.

$f(x) = 2x^2 - 3x - 4$

$$(2) + (-3) + c = -5$$
$$-1 + c = -5$$
$$c = -4$$



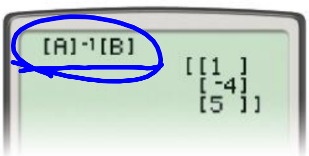
Write the function using $a = 2$, $b = -3$ and $c = -4$.

You may use any method that you studied in Chapters 3 or 4 to solve the system of three equations in three variables. For example, you can use a matrix equation as shown.

$0a + 0b + c = 5$

constant 3×1

$$\begin{cases} c = 5 \\ 4a + 2b + c = 1 \\ 9a + 3b + c = 2 \end{cases} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}$$



[A] [B] $[A]^{-1}[B]$

A **quadratic model** is a quadratic function that represents a real data set. Models are useful for making estimates.

In Chapter 2, you used a graphing calculator to perform a *linear regression* and make predictions. You can apply a similar statistical method to make a quadratic model for a given data set using **quadratic regression**.

Helpful Hint

The coefficient of determination R^2 shows how well a quadratic function model fits the data. The closer R^2 is to 1, the better the fit. In a model with $R^2 \approx 0.996$, which is very close to 1, the quadratic model is a good fit.

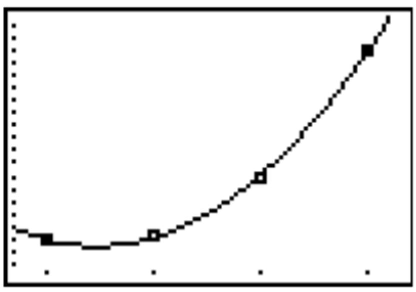
Example 3: Consumer Application

The table shows the cost of circular plastic wading pools based on the pool's diameter. Find a quadratic model for the cost of the pool, given its diameter. Use the model to estimate the cost of the pool with a diameter of 8 ft.

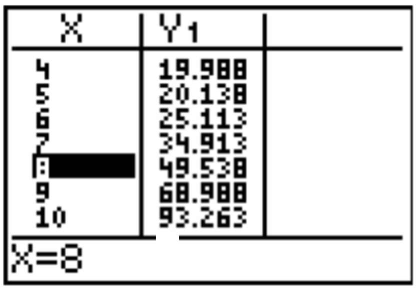
Diameter (ft.)	4	5	6	7
Cost	$19.95	$20.25	$25.00	$34.95

Example 3 Continued

Step 3 Graph the data and function model to verify that the model fits the data.



Step 4 Use the table feature to find the function value $x=8$.



X	Y1
4	19.988
5	20.138
6	25.113
7	34.913
8	49.538
9	68.988
10	93.263

X=8

A quadratic model is $f(x) \approx 2.4x^2 - 21.6x + 67.6$, where x is the diameter in feet and $f(x)$ is the cost in dollars. For a diameter of 8 ft, the model estimates a cost of about $49.54.

Check It Out! Example 3

The tables shows approximate run times for 16 mm films, given the diameter of the film on the reel. Find a quadratic model for the reel length given the diameter of the film. Use the model to estimate the reel length for an 8-inch-diameter film.

Film Run Times (16 mm)

Diameter (in)	Reel Length (ft)	Run Time (min)
5	200	5.55
7	400	11.12
9.25	600	16.67
10.5	800	22.22
12.25	1200	33.33
13.75	1600	44.25

Check It Out! Example 4 Continued

Step 1 Enter the data into two lists in a graphing calculator.

L1	L2	L3 1
5	200	------
7	400	
9.25	600	
10.5	800	
12.25	1200	
13.75	1600	
------	------	

L1(1)=5

Step 2 Use the quadratic regression feature.

QuadReg
y=ax²+bx+c
a=14.30488699
b=-112.4059208
c=430.1099673
R²=.9964519736

Check It Out! Example 4 Continued

A quadratic model is $L(d) \approx 14.3d^2 - 112.4d + 430.1$, where d is the diameter in inches and $L(d)$ is the reel length. For a diameter of 8 in., the model estimates the reel length to be about 446 ft.

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