6-4 Factoring Polynomials

Warm Up

Factor each expression.

1. $3x - 6y$ $3(x - 2y)$

2. $a^2 - b^2$ $(a + b)(a - b)$

Find each product.

3. $(x - 1)(x + 3)$ $x^2 + 2x - 3$

4. $(a + 1)(a^2 + 1)$ $a^3 + a^2 + a + 1$

Objectives

Use the Factor Theorem to determine factors of a polynomial.

Factor the sum and difference of two cubes.

Example 1: Determining Whether a Linear Binomial is a Factor

Determine whether the given binomial is a factor of the polynomial $P(x)$.

A. $(x + 1)$; $(x^2 - 3x + 1)$

Find $P(-1)$ by synthetic substitution.

$$\begin{array}{r|rrr} -1 & 1 & -3 & 1 \\ & & -1 & 4 \\ \hline & 1 & -4 & 5 \end{array}$$

$P(-1) = 5$

$P(-1) \neq 0$, so $(x + 1)$ is not a factor of $P(x) = x^2 - 3x + 1$.

B. $(x + 2)$; $(3x^4 + 6x^3 - 5x - 10)$

Find $P(-2)$ by synthetic substitution.

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -5 & -10 \\ & & -6 & 0 & 0 & 10 \\ \hline & 3 & 0 & 0 & -5 & 0 \end{array}$$

$P(-2) = 0$, so $(x + 2)$ is a factor of $P(x) = 3x^4 + 6x^3 - 5x - 10$.

Check It Out! Example 1

Determine whether the given binomial is a factor of the polynomial $P(x)$.

a. $(x + 2)$; $(4x^2 - 2x + 5)$

Find $P(-2)$ by synthetic substitution.

-2 | $4 \quad -2 \quad 5$

$0 \quad -8 \quad 20$

$4 \quad -10 \quad 25$

$4x - 10$

$+25$

$x+2$

b. $(3x - 6)$; $(3x^4 - 6x^3 + 6x^2 + 3x - 30)$

$3x - 6 = 0$

$+6 \quad +6$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

Divide the polynomial by 3, then find $P(2)$ by synthetic substitution.

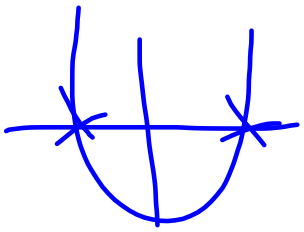
2 | $3 \quad -6 \quad 6 \quad 3 \quad -30$

$0 \quad 6 \quad 0 \quad 12 \quad 30$

$3 \quad 0 \quad 6 \quad 15 \quad | \quad 0$

$3x^3 + 6x + 15$

You are already familiar with methods for factoring quadratic expressions. You can factor polynomials of higher degrees using many of the same methods you learned in Lesson 5-3.



Example 2: Factoring by Grouping

Factor: $x^3 - x^2 - 25x + 25$.

$25(-x+1)$

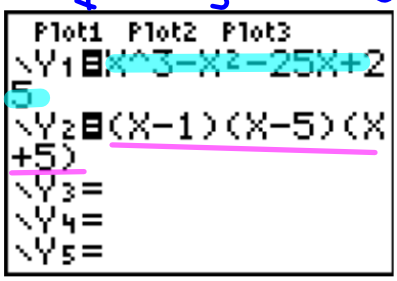
$(x^3 - x^2) + (-25x + 25)$ *Group terms.*

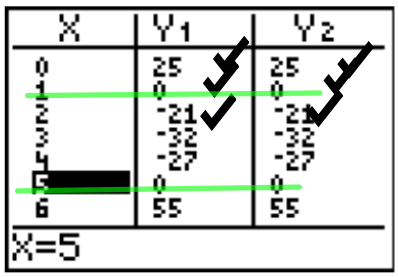
$x^2(x - 1) - 25(x - 1)$ *Factor common monomials from each group.*

$(x - 1)(x^2 - 25)$ *Factor out the common binomial $(x - 1)$.*

$(x - 1)(x - 5)(x + 5)$ *Factor the difference of squares.*

1 5 -5





Check It Out! Example 2a

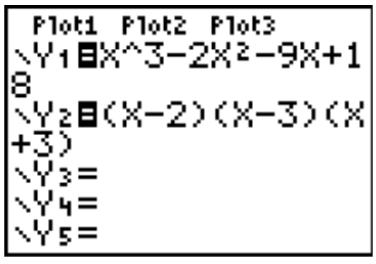
Factor: $x^3 - 2x^2 - 9x + 18$.

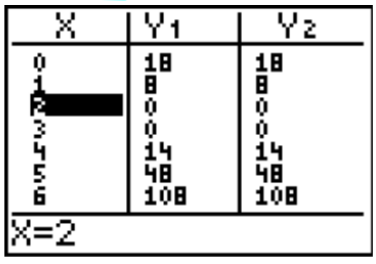
$(x^3 - 2x^2) + (-9x + 18)$ *Group terms.*

$x^2(x - 2) - 9(x - 2)$ *Factor common monomials from each group.*

$(x - 2)(x^2 - 9)$ *Factor out the common binomial $(x - 2)$.*

$(x - 2)(x - 3)(x + 3)$ *Factor the difference of squares.*





Check It Out! Example 2b

Factor: $2x^3 + x^2 + 8x + 4$.

$(x + 2)(x + 2)$

$(2x^3 + x^2) + (8x + 4)$ *Group terms.*

$x^2(2x + 1) + 4(2x + 1)$ *Factor common monomials from each group.*

$(2x + 1)(x^2 + 4)$ *Factor out the common binomial (2x + 1).*

$(2x + 1)(x^2 + 4)$

$x = -\frac{1}{2}$ $x^2 = -4$

$\sqrt{-4} = \pm 2i$

Just as there is a special rule for factoring the difference of two squares, there are special rules for factoring the sum or difference of two cubes.

Factoring the Sum and the Difference of Two Cubes

METHOD	ALGEBRA
Sum of two cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of two cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

p. 431

Example 3A: Factoring the Sum or Difference of Two Cubes

Factor the expression.

$4x^4 + 108x$

$4x(x^3 + 27)$

$4x(x^3 + 3^3)$

$a = x$ $b = 3$

$4x(x + 3)(x^2 - x \cdot 3 + 3^2)$

$4x(x + 3)(x^2 - 3x + 9)$

$125d^3 - 8$

$(5d)^3 - 2^3$

$a = 5d$ $b = 2$

$(5d - 2)[(5d)^2 + 5d \cdot 2 + 2^2]$

$(5d - 2)(25d^2 + 10d + 4)$

Check It Out! Example 3a

Factor the expression.

$8 + z^6$

$(2)^3 + (z^2)^3$

$(2 + z^2)[(2)^2 - 2 \cdot z + (z^2)^2]$

$(2 + z^2)(4 - 2z + z^4)$

$2x^5 - 16x^2$

$2x^2(x^3 - 8)$

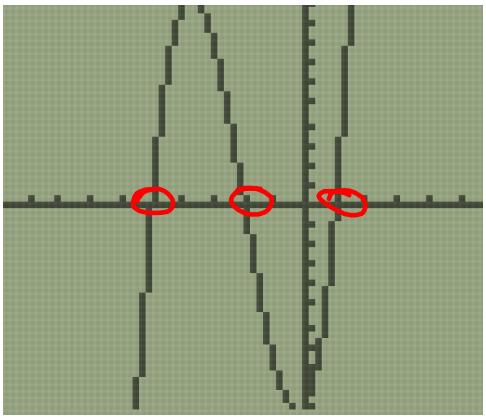
$2x^2(x^3 - 2^3)$

$2x^2(x - 2)(x^2 + x \cdot 2 + 2^2)$

$2x^2(x - 2)(x^2 + 2x + 4)$

Example 4: Geometry Application

The volume of a plastic storage box is modeled by the function $V(x) = x^3 + 6x^2 + 3x - 10$. Identify the values of x for which $V(x) = 0$, then use the graph to factor $V(x)$.



1	1	6	3	−10
		1	7	10
	1	7	10	0

$V(x) = (x - 1)(x^2 + 7x + 10)$

$V(x) = (x - 1)(x + 2)(x + 5)$

$x = -5, -2, 1$

$(x+5)(x+2)(x-1)$

p. 426 # 18, 20, 52

p. 433 # 18- 26 even, 32, 40.

DUE TOMORROW

Check It Out! Example 4

The volume of a rectangular prism is modeled by the function $V(x) = x^3 - 8x^2 + 19x - 12$, which is graphed below. Identify the values of x for which $V(x) = 0$, then use the graph to factor $V(x)$.

Lesson Quiz

1. $x - 1$; $P(x) = 3x^2 - 2x + 5$

2. $x + 2$; $P(x) = x^3 + 2x^2 - x - 2$

3. $x^3 + 3x^2 - 9x - 27$

4. $x^3 + 3x^2 - 28x - 60$

4. $64p^3 - 8q^3$