6-5 Finding Real Roots of Polynomial Equations

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

Factor completely.

x^2-2x-8
$(x-4)(x+2)$

1. $2y^3 + 4y^2 - 30y$ $2y(y-3)(y+5)$

2. $3x^4 - 6x^2 - 24$ $3(x^4-2x^2-8)$ → $3(x^2-4)(x^2+2)$ $3(x-2)(x+2)(x^2+2)$

Solve each equation.

3. $x^2 - 9 = 0$ $x = -3, 3$

4. $x^3 + 3x^2 - 4x = 0$ $x = -4, 0, 1$ ← $x(x+4)(x-1)$

1. Standard form: $3x^5 + 4x^2 - 5$
   Leading coefficient: 3
   Degree: 5
   Terms: 3
   Name: quintic trinomial

2. Standard form: $13x + 7$
   Leading coefficient: 13
   Degree: 1
   Terms: 2
   Name: linear binomial

3. Standard form: $5x^3 + x^2 - 3x + 1$
   Leading coefficient: 5
   Degree: 3
   Terms: 4
   Name: cubic polynomial with 4 terms

4. Standard form: $2x^4 - 5x^3 + 8x$
   Leading coefficient: 2
   Degree: 4
   Terms: 3
   Name: quartic trinomial

5. $= 7x^2 + 4$

6. $= 7x^3 - 7x^2 - 2$

7. $= 6x^3 + 15x^2 - 2x + 7$

8. $= x^5 + 3x^4 - 10$

9. $998,515.

10. From left to right, it alternately increases and decreases, changing directions twice and crossing the x-axis twice. There appear to be 2 real zeros.

11. From left to right, it alternately increases and decreases, changing direction twice and crossing the x-axis 3 times. There appear to be 3 real zeros.

12. From left to right, its increases then decreases, and never crosses the x-axis. There appear to be no real zeros.

13. $= 8x^2y + 14xy^2$

14. $= 3a^2b + 4ab^2 + b^3$

15. $= 4x^2 + \frac{4}{3}x + \frac{1}{9}$

16. $2x^4 - 5x^3 + 9x^2 + \quad x - 15$

19. $= 256x^4 - 256x^3 + 96x^2 - 16x + 1$

20. $= 6x^3 - 21x^2 - 12x$

21. $= 3y + 8$

22. $= 3x^2 + 2x + 5$

25. $= 3t(t^2 - 7t - 4)$

26. $= (4y - 7)(4y + 7)$

27. $= (y + 7)(y^2 + 2)$

28. $= (a^2 + 5)(a^4 - 5a^2 + 25)$

29. $V(x)$ has 3 real zeros at $x = -4, -1, 3$. The corresponding factors are $(x + 4)$, $(x + 1)$, and $(x - 3)$.

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60.a. $= \frac{19 \pm 5\sqrt{17}}{32}$

$t \approx -0.05$ or $t \approx 1.2$

b. No,

46. $x = 4$ or $x = -4$

47. $x = 1.5$ or $x = -2.5$

48. $= -1 \pm \sqrt{3}$

50. $= \frac{1 \pm \sqrt{2}}{2}$

36. $= -1.75$ or 1.75
The walker will fall for 1.75 s.

18. $= 3$ or $\frac{1}{3}$

20. $= 4$ or -1

22. $= \frac{7 \pm \sqrt{113}}{4}$

24. $= \frac{-1 \pm i\sqrt{3}}{2}$

29. $= \frac{2 \pm 2i\sqrt{2}}{3}$

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Objectives

Identify the multiplicity of roots.

Use the Rational Root Theorem and the irrational Root Theorem to solve polynomial equations.

Vocabulary

multiplicity

In Lesson 6-4, you used several methods for factoring polynomials. As with some quadratic equations, factoring a polynomial equation is one way to find its real roots.

Recall the Zero Product Property from Lesson 5-3. You can find the *roots*, or *solutions*, of the polynomial equation $P(x) = 0$ by setting each factor equal to 0 and solving for x.

Example 1A: Using Factoring to Solve Polynomial Equations

Solve the polynomial equation by factoring.

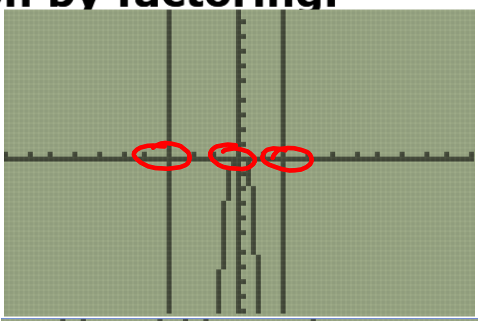
$$4x^6 + 4x^5 - 24x^4 = 0$$

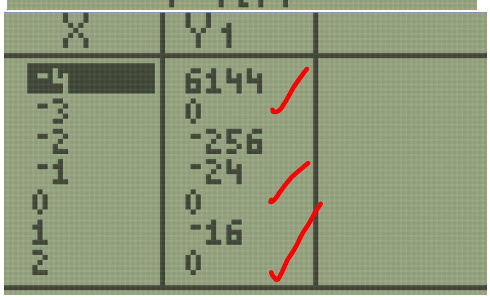
$4x^4(x^2 + x - 6) = 0$

$4x^4(x + 3)(x - 2) = 0$

$4x^4 = 0$ or $(x + 3) = 0$ or $(x - 2) = 0$

$x = 0, x = -3, x = 2$





Example 1B: Using Factoring to Solve Polynomial Equations

Solve the polynomial equation by factoring.

$x^4 + 25 = 26x^2$

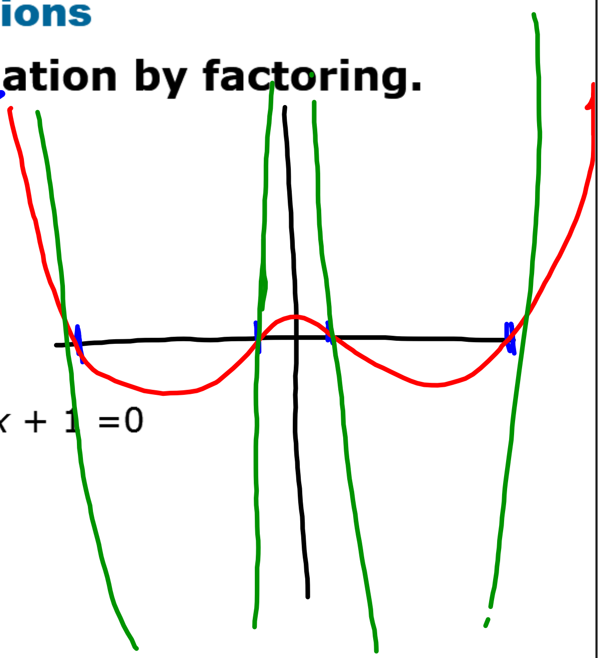
$x^4 - 26x^2 + 25 = 0$

$(x^2 - 25)(x^2 - 1) = 0$

$(x - 5)(x + 5)(x - 1)(x + 1)$

$x - 5 = 0,\ x + 5 = 0,\ x - 1 = 0,$ or $x + 1 = 0$

$x = 5,\ x = -5,\ x = 1$ or $x = -1$



Check It Out! Example 1a

Solve the polynomial equation by factoring.

$2x^6 - 10x^5 - 12x^4 = 0$

$2x^4(x^2 - 5x - 6)$

$2x^4(x - 6)(x + 1)$

$0 \quad 6 \quad -1$

$x^3 - 2x^2 - 25x = -50$

$x^3 - 2x^2 - 25x + 50 =$

$(x^3 - 2x^2)(-25x + 50)$

$x^2(x - 2) - 25(x - 2)$

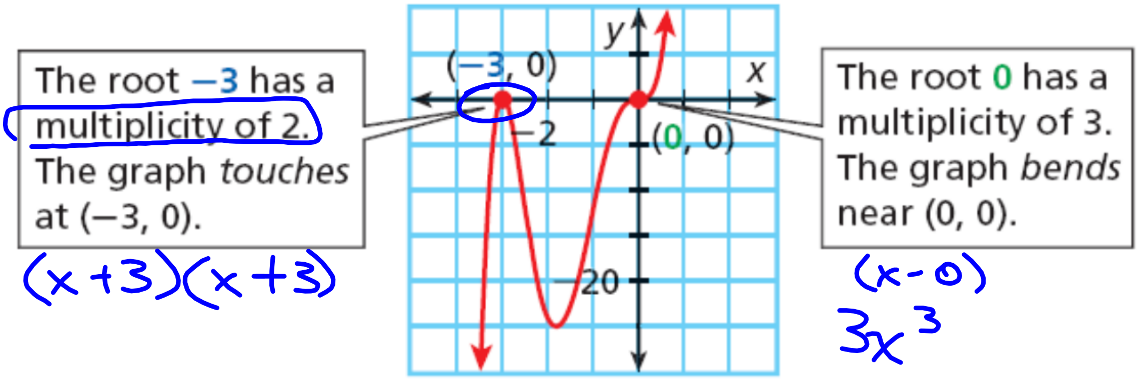
$(x^2 - 25)(x - 2)$

$(x + 5)(x - 5)(x - 2)$

$\pm 5,\ 2$

Sometimes a polynomial equation has a factor that appears more than once. This creates a *multiple root*. In $3x^5 + 18x^4 + 27x^3 = 0$ has two multiple roots, 0 and −3. For example, the root 0 is a factor three times because $3x^3 = 0$.

The **multiplicity** of root r is the number of times that $x - r$ is a factor of $P(x)$. When a real root has even multiplicity, the graph of $y = P(x)$ touches the x-axis but does not cross it. When a real root has odd multiplicity greater than 1, the graph "bends" as it crosses the x-axis.



$(x+3)(x+3)$

$(x-0)$

$3x^3$

$3x^3(x+3)(x+3)$

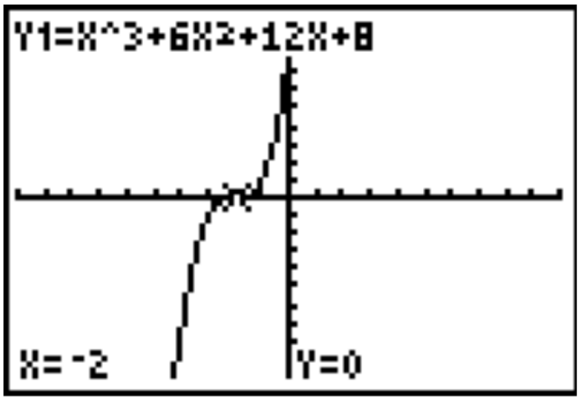
You cannot always determine the multiplicity of a root from a graph. It is easiest to determine multiplicity when the polynomial is in factored form.

Example 2A: Identifying Multiplicity

Identify the roots of each equation. State the multiplicity of each root.
$x^3 + 6x^2 + 12x + 8 = 0$

$x^3 + 6x^2 + 12x + 8 = (x + 2)(x + 2)(x + 2)$

$x + 2$ is a factor three times. The root −2 has a multiplicity of 3.



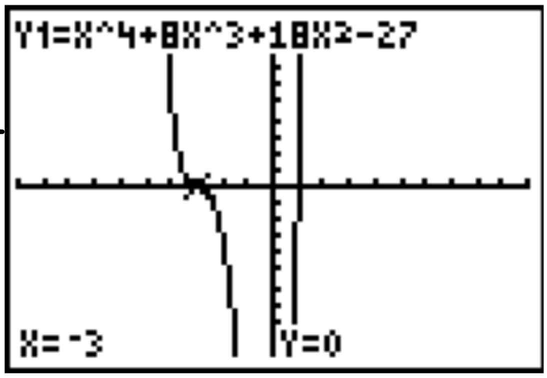
Check Use a graph. A calculator graph shows a bend near (−2, 0). ✓

Example 2B: Identifying Multiplicity

Identify the roots of each equation. State the multiplicity of each root.
$x^4 + 8x^3 + 18x^2 – 27 = 0$

$x^4 + 8x^3 + 18x^2 – 27 = (x – 1)(x + 3)(x + 3)(x + 3)$

$x – 1$ is a factor once, and $x + 3$ is a factor three times. The root 1 has a multiplicity of 1. The root –3 has a multiplicity of 3.



Check Use a graph. A calculator graph shows a bend near (–3, 0) and crosses at (1, 0). ✓

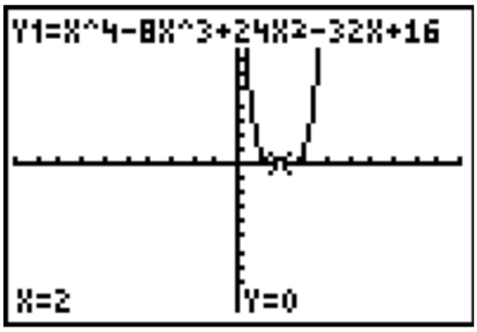
Check It Out! Example 2a

Identify the roots of each equation. State the multiplicity of each root.
$x^4 - 8x^3 + 24x^2 - 32x + 16 = 0$

$x^4 - 8x^3 + 24x^2 - 32x + 16 = (x - 2)(x - 2)(x - 2)(x - 2)$

$x - 2$ is a factor four times. The root 2 has a multiplicity of 4.

Check Use a graph. A calculator graph shows a bend near (2, 0). ✓



Check It Out! Example 2b

Identify the roots of each equation. State the multiplicity of each root.
$2x^6 - 22x^5 + 48x^4 + 72x^3 = 0$

$2x^6 - 22x^5 + 48x^4 + 72x^3 = 2x^3(x + 1)(x - 6)(x - 6)$

x is a factor three times, $x + 1$ is a factor once, and $x - 6$ is a factor two times.

The root 0 has a multiplicity of 3. The root –1 has a multiplicity of 1. The root 6 has a multiplicity of 2.

Not all polynomials are factorable, but the Rational Root Theorem can help you find all possible rational roots of a polynomial equation.

Rational Root Theorem

If the polynomial $P(x)$ has integer coefficients, then every rational root of the polynomial equation $P(x) = 0$ can be written in the form $\frac{p}{q}$, where p is a factor of the constant term of $P(x)$ and q is a factor of the leading coefficient of $P(x)$.

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$2x^2 - 5x + 2$ (q: 2, p: 2)

$\frac{\pm 1, 2}{\pm 1, 2}$ → $\pm 1, 2, \frac{1}{2}$

$(2x-1)(x-2)$

$2x^2 - 4x - x + 2$

$2x^2 - 5x + 2$

Synthetic division by 2:

2	2	-5	2
		4	-2
	2	-1	0

$9x^2 + 12x + 4 \quad \pm \frac{1, 2, 4}{1, 3, 9} =$

$\pm 1, 2, 4, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{1}{9}, \frac{2}{9}, \frac{4}{9}$

$-\frac{2}{3}$ | 9 12 4

$-\frac{18}{3} = -6 \quad -\frac{12}{3} = -4$

9 6 | 0

$(x+\frac{2}{3})(9x+6)$

$(x+\frac{2}{3})(x+\frac{6}{9})$

$(x+\frac{2}{3})(x+\frac{2}{3})$

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Example 3: Marketing Application

The design of a box specifies that its length is 4 inches greater than its width. The height is 1 inch less than the width. The volume of the box is 12 cubic inches. What is the width of the box?

$\pm \frac{1, 2, 3, 4, 6, 12}{1}$

$w = x$

$L = x + 4$

$h = x - 1$

$x(x+4)(x-1) = 12$

$x(x^2+3x-4) = 12$

$x^3 + 3x^2 - 4x - 12 = 0$

2 | 1 3 -4 -12

0 1 4 0

1 4 0 | -12

$(x-2)(x+3)(x+2)$

$(x-2)(x^2+5x+6)$

$\frac{p}{q}$	1	3	−4	−12
1	1	4	0	−12
2	1	5	6	0
3	1	6	14	30
4	1	7	24	84

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Example 3 Continued

$(x - 2)(x^2 + 5x + 6) = 0$ *Set the equation equal to 0.*

$(x - 2)(x + 2)(x + 3) = 0$ *Factor $x^2 + 5x + 6$.*

$x = 2, x = -2$, or $x = -3$ *Set each factor equal to 0, and solve.*

The width must be positive, so the width should be 2 inches.

Polynomial equations may also have irrational roots.

Irrational Root Theorem

If the polynomial $P(x)$ has rational coefficients and $a + b\sqrt{c}$ is a root of the polynomial equation $P(x) = 0$, where a and b are rational and $\sqrt{c}$ is irrational, then $a - b\sqrt{c}$ is also a root of $P(x) = 0$.

The Irrational Root Theorem say that irrational roots come in conjugate pairs. For example, if you know that $1 + \sqrt{2}$ is a root of $x^3 - x^2 - 3x - 1 = 0$, then you know that $1 - \sqrt{2}$ is also a root.

Recall that the real numbers are made up of the rational and irrational numbers. You can use the Rational Root Theorem and the Irrational Root Theorem together to find *all* of the real roots of $P(x) = 0$.

Example 4: Identifying All of the Real Roots of a Polynomial Equation

Identify all the real roots of $2x^3 - 9x^2 + 2 = 0$.

Step 1 Use the Rational Root Theorem to identify possible rational roots.

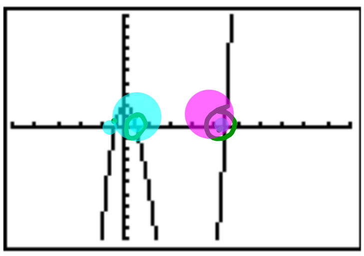
p = 2 and q = 2

$$\frac{\pm 1, \pm 2}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm \frac{1}{2}.$$

$\frac{1}{2}$ $\left(x - \frac{1}{2}\right)$

Step 2 Graph $y = 2x^3 - 9x^2 + 2$ to find the x-intercepts.

The x-intercepts are located at or near –0.45, 0.5, and 4.45. The x-intercepts –0.45 and 4.45 do not correspond to any of the possible rational roots.



Example 4 Continued

Step 3 Test the possible rational root $\frac{1}{2}$.

$\frac{1}{2}$	2	−9	0	2
		1	−4	2
	2	−8	−4	0

Test $\frac{1}{2}$. The remainder is 0, so $(x - \frac{1}{2})$ is a factor.

The polynomial factors into $(x - \frac{1}{2})(2x^2 - 8x - 4)$.

Step 4 Solve $2x^2 - 8x - 4 = 0$ to find the remaining roots.

$2(x^2 - 4x - 2) = 0$ *Factor out the GCF, 2*

$x = \frac{4 \pm \sqrt{16+8}}{2} = 2 \pm \sqrt{6}$ *Use the quadratic formula to identify the irrational roots.*

Example 4 Continued

The fully factored equation is

$$2\left(x - \frac{1}{2}\right)\left[x - \left(2 + \sqrt{6}\right)\right]\left[x - \left(2 - \sqrt{6}\right)\right] = 0$$

The roots are $\frac{1}{2}$, $2 + \sqrt{6}$, and $2 - \sqrt{6}$.

Homework

Solve by factoring.

1. $x^3 + 9 = x^2 + 9x$

Identify the roots of each equation. State the multiplicity of each root.

2. $5x^4 - 20x^3 + 20x^2 = 0$

3. $x^3 - 12x^2 + 48x - 64 = 0$

4. A box is 2 inches longer than its height. The width is 2 inches less than the height. The volume of the box is 15 cubic inches. How tall is the box?

5. Identify all the real roots of $x^3 + 5x^2 - 3x - 3 = 0$.

Check It Out! Example 4

Identify all the real roots of $2x^3 - 3x^2 - 10x - 4 = 0$.

Check It Out! Example 4 Continued

Step 3 Test the possible rational root $-\frac{1}{2}$.

$$-\frac{1}{2}\ \Big|\ \ 2 \quad -3 \quad -10 \quad -4$$

Test $-\frac{1}{2}$. The remainder is 0, so $(x + \frac{1}{2})$ is a factor.

Check It Out! Example 4 Continued

The fully factored equation is

$$2\left(x + \frac{1}{2}\right)\left[x - \left(1 + \sqrt{5}\right)\right]\left[x - \left(1 - \sqrt{5}\right)\right]$$

The roots are $-\frac{1}{2}$, $1 + \sqrt{5}$, and $1 - \sqrt{5}$.

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