7-1 Exponential Functions, Growth and Decay

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

Evaluate.

1. $100(1.08)^{20}$ ≈ 466.1

2. $100(0.95)^{25}$ ≈ 27.74

3. $100(1 - 0.02)^{10}$ ≈ 81.71

4. $100(1 + 0.08)^{-10}$ ≈ 46.32

Objective

Write and evaluate exponential expressions to model growth and decay situations.

Vocabulary

exponential function
base
asymptote
exponential growth
exponential decay

Moore's law, a rule used in the computer industry, states that the number of transistors per integrated circuit (the processing power) doubles every year. Beginning in the early days of integrated circuits, the growth in capacity may be approximated by this table.

Transistors per Integrated Chip

Year	1965	1966	1967	1968	1969	1970	1971
Transistors	60	120	240	480	960	1920	3840

×2 ×2 ×2 ×2 ×2 ×2

Growth that doubles every year can be modeled by using a function with a variable as an exponent. This function is known as an *exponential function*. The parent **exponential function** is $f(x) = b^x$, where the **base** b is a constant and the exponent x is the independent variable.

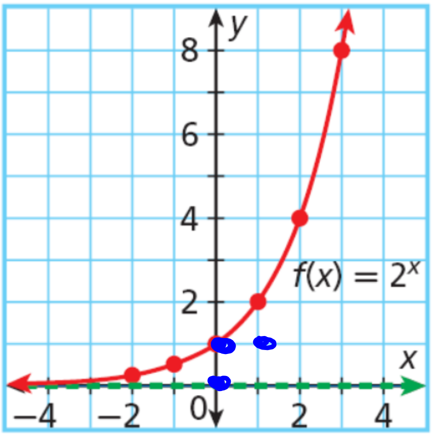
Base Exponent

$$f(x) = b^x, \text{ where } b > 0, b \neq 1$$

1

The graph of the parent function $f(x) = 2^x$ is shown. The domain is all real numbers and the range is $\{y | y > 0\}$.

$2^0 = 1$



x	–2	–1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

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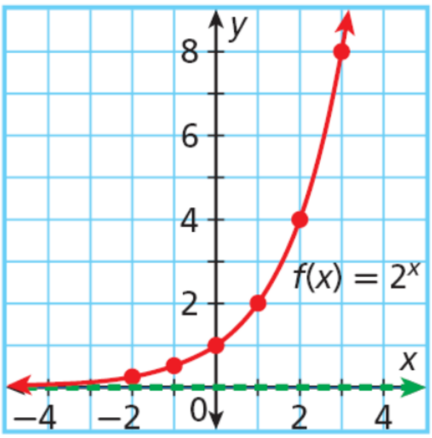
Notice as the x-values decrease, the graph of the function gets closer and closer to the x-axis. The function never reaches the x-axis because the value of 2^x cannot be zero. In this case, the x-axis is an *asymptote*. An **asymptote** is a line that a graphed function approaches as the value of x gets very large or very small.



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A function of the form $f(x) = ab^x$, with $a > 0$ and $b > 1$, is an **exponential growth** function, which increases as x increases. When $0 < b < 1$, the function is called an **exponential decay** function, which decreases as x increases.

$$\frac{x^1}{x^1} = x^{1-1} = x^0 = 1$$

Remember!

In the function $y = b^x$, y is a function of x because the value of y *depends* on the value of x.

$$\frac{x y^{-2} z^3}{4} = \frac{x z^3}{4y^2} \qquad \frac{x^{-2}}{y^{-3}} = \frac{y^3}{x^2}$$

Remember!

Negative exponents indicate a reciprocal. For example:

$$x^{-2} = \frac{1}{x^2} \qquad \frac{2}{x^{-3}} = 2x^3 \qquad 2x^{-2} = \frac{2}{x^2}$$

Example 1A: Graphing Exponential Functions

Tell whether the function shows growth or decay. Then graph.

$f(x) = 10\left(\frac{3}{4}\right)^x$

10(1) a=10 b= 3/4

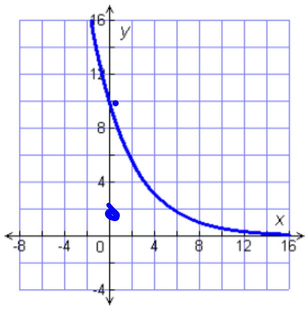
Step 1 Find the value of the base.

$f(x) = 10\left(\frac{3}{4}\right)^x$

The base , $\frac{3}{4}$,is less than 1. This is an exponential decay function.

Step 2 Graph the function by using a table of values.

x	0	2	4	6	8	10	12
f(x)	10	5.6	3.2	1.8	1.0	0.6	0.3



Example 1B: Graphing Exponential Functions

Tell whether the function shows growth or decay. Then graph.

$g(x) = 100(1.05)^x$

Step 1 Find the value of the base.

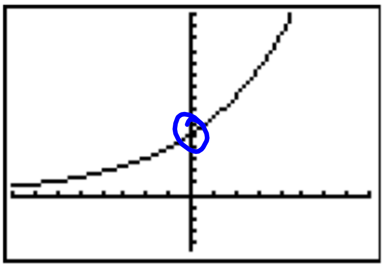
$g(x) = 100(1.05)^x$

100(1)

g(0)= 100

The base, 1.05, is greater than 1. This is an exponential growth function.

Step 2 Graph the function by using a graphing calculator.



Check It Out! Example 1

Tell whether the function $p(x) = 5(1.2^x)$ shows growth or decay. Then graph.

$5(1.2)^x$

base = 1.2

exp. growth

You can model growth or decay by a constant percent increase or decrease with the following formula:

Initial amount — Number of time periods

$$A(t) = a(1 \pm r)^t$$

Final amount — Rate of increase

12 / 364.25

12%

In the formula, the base of the exponential expression, $1 + r$, is called the *growth factor.* Similarly, $1 - r$ is the *decay factor*.

Helpful Hint

X is used on the graphing calculator for the variable *t:* **Y1 =5000*1.0625^X**

Example 2: Economics Application

Clara invests $5000 in an account that pays 6.25% interest per year. After how many years will her investment be worth $10,000?

Step 1 Write a function to model the growth in value of her investment.

$f(t) = a(1 + r)^t$

$f(t) = 5000(1 + 0.0625)^t$

$f(t) = 5000(1.0625)^t$

$10,000 = 5000(1.0625)^x$

$5,000(1.0625)^x - 10,000$

Check It Out! Example 2

In 1981, the Australian humpback whale population was 350 and increased at a rate of 14% each year since then. Write a function to model population growth. Use a graph to predict when the population will reach 20,000.

$P(t) = a(1 + r)^t$

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Example 3: Depreciation Application

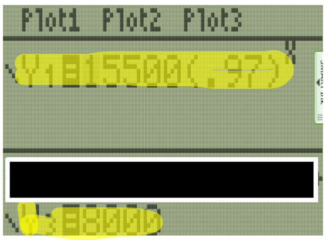
A city population, which was initially 15,500, has been dropping 3% a year. Write an exponential function and graph the function. Use the graph to predict when the population will drop below 8000.

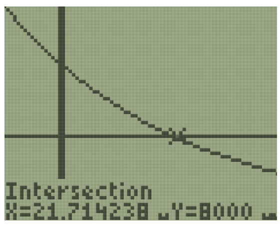
y=8,000

$f(t) = a(1 - r)^t$

$f(t) = 15{,}500(1 - 0.03)^t$

$f(t) = 15{,}500(0.97)^t$





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Check It Out! Example 3

A motor scooter purchased for $1000 depreciates at an annual rate of 15%. Write an exponential function and graph the function. Use the graph to predict when the value will fall below $100.

$f(t) = a(1 - r)^t$

Lesson Quiz

In 2000, the world population was 6.08 billion and was increasing at a rate 1.21% each year.

1. Write a function for world population. Does the function represent growth or decay?

2. Use a graph to predict the population in 2020.

The value of a $3000 computer decreases about 30% each year.

3. Write a function for the computer's value. Does the function represent growth or decay?

4. Use a graph to predict the value in 4 years.

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