7-2 Inverses of Relations and Functions

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

Solve for *y*.

1. $x = 3y - 7$ $y = \frac{x + 7}{3}$

2. $x = \frac{y + 5}{8}$ $y = 8x - 5$

3. $x = 4 - y$ $y = 4 - x$

4. $x = y^2$ $y = \pm\sqrt{x}$

Objectives

Graph and recognize inverses of relations and functions.

Find inverses of functions.

Vocabulary

inverse relation

inverse function

You have seen the word *inverse* used in various ways.

The additive inverse of 3 is –3.

The multiplicative inverse of 5 is $\frac{1}{5}$.

The multiplicative inverse matrix of

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \text{ is } A^{-1} = \begin{bmatrix} 1 & -0.5 \\ -2 & 1.5 \end{bmatrix}.$$

You can also find and apply inverses to relations and functions. To graph the **inverse relation**, you can reflect each point across the line $y = x$. This is equivalent to switching the x- and y-values in each ordered pair of the relation.

Remember!

A *relation* is a set of ordered pairs. A *function* is a relation in which each x-value has, at most, one y-value paired with it.

Example 1: Graphing Inverse Relations

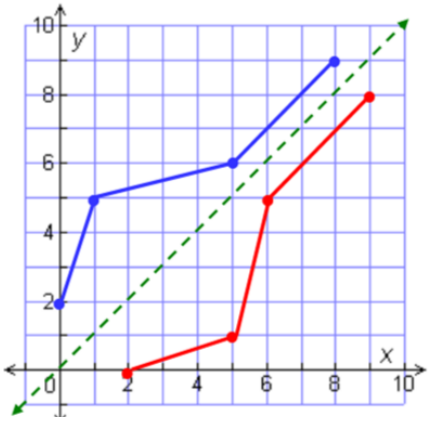
Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

x	0	1	5	8
y	2	5	6	9

Graph each ordered pair and connect them.

Switch the *x*- and *y*-values in each ordered pair.

x	2	5	6	9
y	0	1	5	8



Domain: $\{x \mid 0 \le x \le 8\}$ Range : $\{y \mid 2 \le x \le 9\}$

Domain: $\{x \mid 2 \le x \le 9\}$ Range : $\{y \mid 0 \le x \le 8\}$

Check It Out! Example 1

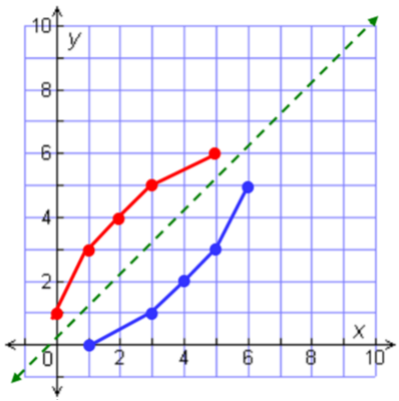
Graph the relation and connect the points. Then graph the inverse. Identify the domain and range of each relation.

x	1	3	4	5	6
y	0	1	2	3	5

Graph each ordered pair and connect them.

Switch the *x*- and *y*-values in each ordered pair.

x	0	1	2	3	5
y	1	3	4	5	6

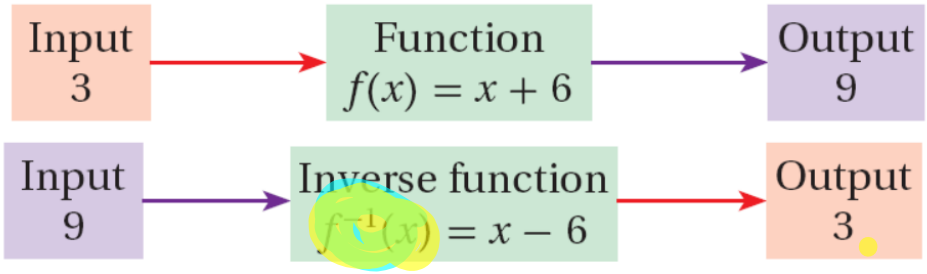


Domain: $\{1 \le x \le 6\}$ Range : $\{0 \le y \le 5\}$

Domain: $\{0 \le y \le 5\}$ Range : $\{1 \le x \le 6\}$

When the relation is also a function, you can write the inverse of the function $f(x)$ as $f^{-1}(x)$. This notation does *not* indicate a reciprocal.

Functions that undo each other are **inverse functions**.



To find the inverse function, use the inverse operation. In the example above, 6 is added to x in $f(x)$, so 6 is subtracted to find $f^{-1}(x)$.

Example 2: Writing Inverses of by Using Inverse Functions

Use inverse operations to write the inverse of $f(x) = x - \frac{1}{2}$.

$f(x) = x - \frac{1}{2}$

$f^{-1}(x) = x + \frac{1}{2}$

Check Use the input $x = 1$ in $f(x)$.

$f(x) = x - \frac{1}{2}$

$f(1) = 1 - \frac{1}{2}$

$= \frac{1}{2}$

Substitute the result into $f^{-1}(x)$

$f^{-1}(x) = x + \frac{1}{2}$

$f^{-1}(\frac{1}{2}) = \frac{1}{2} + \frac{1}{2}$

$= 1$

Check It Out! Example 2a

Use inverse operations to write the inverse of $f(x) = \frac{x}{3}$.

$f(x) = \frac{x}{3}$

$f^{-1}(x) = 3x$

Check Use the input $x = 1$ in $f(x)$.

$f(x) = \frac{x}{3}$

$f(1) = \frac{1}{3}$

$= \frac{1}{3}$

Substitute the result into $f^{-1}(x)$

$f^{-1}(x) = 3x$

$f^{-1}(\frac{1}{3}) = 3(\frac{1}{3})$

$= 1$

Undo operations in the opposite order of the order of operations.

Helpful Hint

The *reverse* order of operations:
Addition or Subtraction
Multiplication or Division
Exponents
Parentheses

Example 3: Writing Inverses of Multi-Step Functions

$y = \frac{x}{3} + 7$

Use inverse operations to write the inverse of $f(x) = 3(x - 7)$.

$f(x) = 3(x - 7)$ *The variable x is subtracted by 7, then is multiplied by 3.*

$f^{-1}(x) = \frac{1}{3}x + 7$ *First, undo the multiplication by dividing by 3. Then, undo the subtraction by adding 7.*

Check Use a sample input.

$f(9) = 3(9 - 7) = 3(2) = 6 \quad f^{-1}(6) = \frac{1}{3}(6) + 7 = 2 + 7 = 9$ ✓

Check It Out! Example 3

Use inverse operations to write the inverse of $f(x) = 5x - 7$.

$f(x) = 5x - 7.$ *The variable x is multiplied by 5, then 7 is subtracted.*

$f^{-1}(x) = \frac{x + 7}{5}$ *First, undo the subtraction by adding by 7. Then, undo the multiplication by dividing by 5.*

Check Use a sample input.

$f(2) = 5(2) - 7 = 3 \quad f^{-1}(3) = \frac{3 + 7}{5} = \frac{10}{5} = 2$ ✓

You can also find the inverse function by writing the original function with x and y switched and then solving for y.

Example 4: Writing and Graphing Inverse Functions

Graph $f(x) = -\frac{1}{2}x - 5$. Then write the inverse and graph.

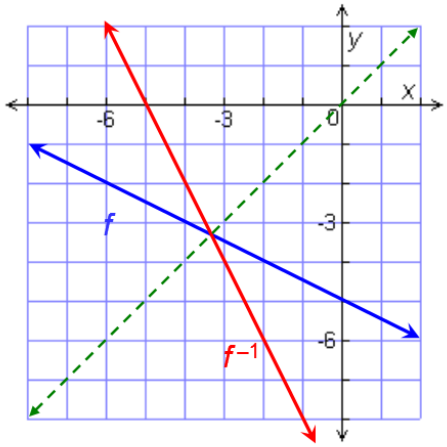
$$y = -\frac{1}{2}x - 5$$

$$x = -\frac{1}{2}y - 5$$

$$(-2)(x + 5) = -\frac{1}{2}y\ (-2)$$

$$y = -2(x + 5)$$

$$y = -2x - 10$$



Check It Out! Example 4

Graph $f(x) = \frac{2}{3}x + 2$. Then write the inverse and graph.

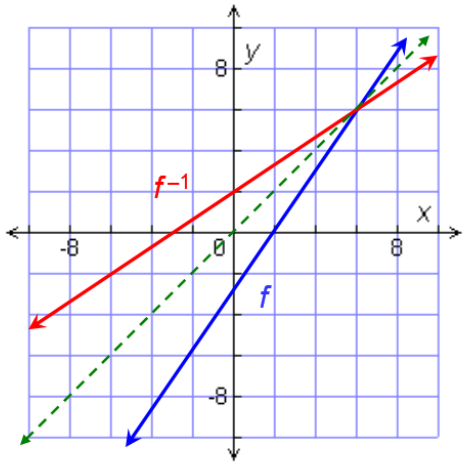
$$y = \frac{2}{3}x + 2$$

$$x = \frac{2}{3}y + 2$$

$$x - 2 = \frac{2}{3}y$$

$$3x - 6 = 2y$$

$$\frac{3}{2}x - 3 = y$$



Anytime you need to undo an operation or work backward from a result to the original input, you can apply inverse functions.

Remember!

In a real-world situation, don't switch the variables, because they are named for specific quantities.

Example 5: Retailing Applications

Juan buys a CD online for 20% off the list price. He has to pay $2.50 for shipping. The total charge is $13.70. What is the list price of the CD?

100 − 20 = 80

Step 1 Write an equation for the total charge as a function of the list price.

$c = 0.80L + 2.50$ *Charge c is a function of list price L.*

Step 2 Find the inverse function that models list price as a function of the change.

$c - 2.50 = 0.80L$ *Subtract 2.50 from both sides.*

$\frac{c - 2.50}{0.80} = L$ *Divide to isolate L.*

Example 5 Continued

Step 3 Evaluate the inverse function for $c = \$13.70$.

$L = \frac{13.70 - 2.50}{0.80}$ *Substitute 13.70 for c.*

$= 14$

The list price of the CD is $14.

Check $c = 0.80L + 2.50$

$= 0.80(14) + 2.50$ *Substitute.*

$= 11.20 + 2.50$

$= 13.70$ ✓

Check It Out! Example 5

To make tea, use $\frac{1}{6}$ teaspoon of tea per ounce of water plus a teaspoon for the pot. Use the inverse to find the number of ounces of water needed if 7 teaspoons of tea are used.

Step 1 Write an equation for the number of ounces of water needed.

$t = \frac{1}{6}z + 1$

Tea t is a function of ounces of water needed z.

Check It Out! Example 5 Continued

Step 3 Evaluate the inverse function for $t = 7$.

$z = 6(7) - 6 = 36$

36 ounces of water should be added.

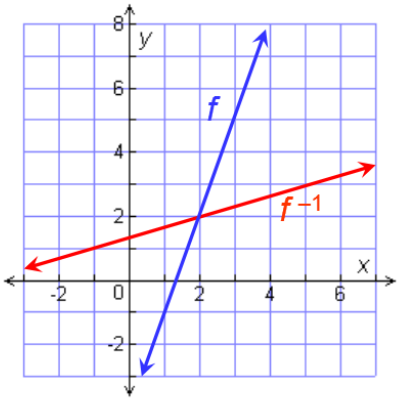
Lesson Quiz: Part I

1. A relation consists of the following points and the segments drawn between them. Find the domain and range of the *inverse* relation:

x	0	3	4	6	9
y	1	2	5	7	8

Lesson Quiz: Part II

2. Graph $f(x) = 3x - 4$. Then write and graph the inverse.



7-2 Inverses of Relations and Functions

Lesson Quiz: Part III

3. A thermometer gives a reading of 25° C. Use the formula $C = \frac{5}{9}(F - 32)$. Write the inverse function and use it to find the equivalent temperature in °F.

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