

7-5**Exponential and Logarithmic Equations and Inequalities**Warm UpLesson PresentationLesson Quiz**Example 3D: Solving Logarithmic Equations****Solve.**

$$\log_{12} x + \log_{12}(x + 1) = 1$$

$$\div \frac{2}{3} = \times \frac{3}{2}$$

$$\log_{12} x(x + 1) = 1$$

Product Property of Logarithms.

$$12^{\log_{12} x(x+1)} = 12^1$$

Exponential form.

$$x(x + 1) = 12$$

Use the inverse properties.

Example 3 Continued

$$x^2 + x - 12 = 0$$

Multiply and collect terms.

$$(x - 3)(x + 4) = 0$$

Factor.

$$-2^3 = -8$$

$$x - 3 = 0 \text{ or } x + 4 = 0$$

Set each of the factors equal to zero.

$$x = 3 \text{ or } x = -4$$

Solve.

Check Check both solutions in the original equation.

$$\log_{12} x + \log_{12}(x + 1) = 1$$

$\log_{12} 3 + \log_{12}(3 + 1)$	1
$\log_{12} 3 + \log_{12} 4$	1
$\log_{12} 12$	1
1	1 ✓

$$\log_{12} x + \log_{12}(x + 1) = 1$$

$\log_{12}(-4) + \log_{12}(-4 + 1)$	1 ✗
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$\log_{12}(-4)$ is undefined.

The solution is $x = 3$.

Check It Out! Example 3a

Solve.

$$3 = \log 8 + 3 \log x$$

$$y_1 = \log 8 + \log x - 3$$

$$3 = \log 8 + 3 \log x$$

$$3 = \log 8 + \log x^3$$

Power Property of Logarithms.

$$3 = \log (8x^3)$$

Product Property of Logarithms.

$$10^3 = 10^{\log (8x^3)}$$

Use 10 as the base for both sides.

$$1000 = 8x^3$$

Use inverse properties on the right side.

$$125 = x^3$$

$$5 = x$$

Check It Out! Example 3b**Solve.**

$$2 \log x - \log 4 = 0$$

$$2 \log \left(\frac{x}{4} \right) = 0$$

$$2(10^{\log \frac{x}{4}}) = 10^0$$

$$\cancel{2} \left(\frac{x}{4} \right) = \cancel{1}^0$$

$$x = 2$$

$$\log x^2 - \log 4 = 0$$

$$\cancel{10} \log \frac{x^2}{4} = \cancel{10}^0$$

$$\frac{x^2}{4} = 1$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\boxed{x = 2}$$

Caution

Watch out for calculated solutions that are not solutions of the original equation.

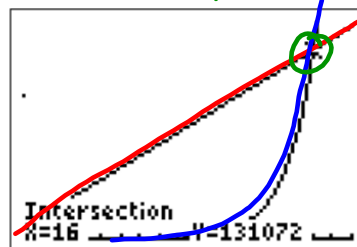
Example 4A: Using Tables and Graphs to Solve Exponential and Logarithmic Equations and Inequalities

Use a table and graph to solve $2^{x+1} > 8192x$.

Use a graphing calculator. Enter $2^{(x+1)}$ as **Y1** and $8192x$ as **Y2**.

X	Y1	Y2
12	8192	98304
13	16384	106496
14	32768	114688
15	65536	122880
16	131072	131072
17	262144	139264
18	524288	147456

X=16



In the table, find the x -values where $Y1$ is greater than $Y2$.

In the graph, find the x -value at the point of intersection.

The solution set is $\{x \mid x > 16\}$.

Example 4B

$$\log(x + 70) = 2\log\left(\frac{x}{3}\right)$$

$$\log(x+70) = \log\left(\frac{x}{3}\right)^2$$

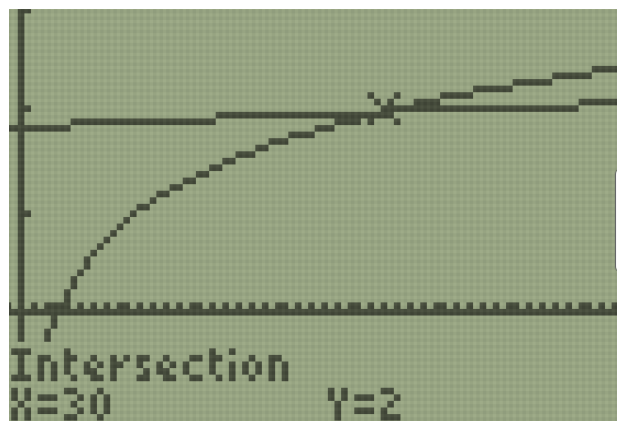
$$x+70 = \left(\frac{x}{3}\right)^2$$

$$9(x+70) = \frac{x^2}{9} \cdot 9$$

$$9x + 630 = x^2$$

$$x^2 - 9x - 630 = 0$$

$$(x-30)(x+21)$$



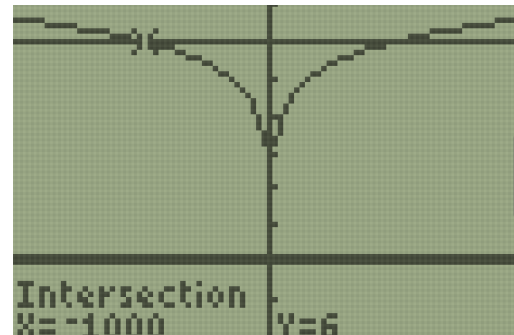
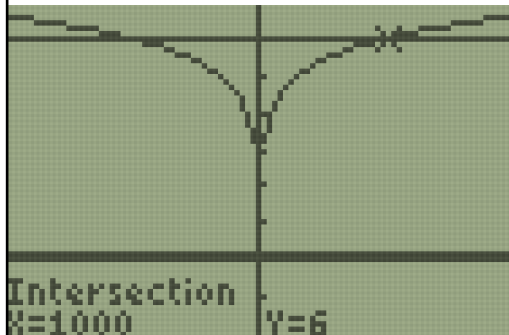
$$\underline{\underline{x=30}}$$

Check It Out! Example 4b

Use a table and graph to solve $2^x > 4^x - 1$.

Check It Out! Example 4c

Use a table and graph to solve $\log x^2 = 6$.



Lesson Quiz: Part I

Solve.

1. $4^{3x-1} = 8^{x+1}$

2. $3^{2x-1} = 20$

3. $\log_7(5x + 3) = 3$

4. $\log(3x + 1) - \log 4 = 2$

5. $\log_4(x - 1) + \log_4(3x - 1) = 2$

1. $4^{3x-1} = 8^{x+1}$

$$2^{2(3x-1)} = 2^{3(x+1)}$$

$$6x - 2 = 3x + 3$$

$$3x = 5$$

$$x = \frac{5}{3}$$

$$\frac{1}{9} = \frac{1}{3^2}$$

3
-2

$$2. 3^{2x-1} = 20$$

$$\log_3 20 + 1$$

$$\log 3^{2x-1} = \log 20$$

$$\frac{(2x-1)\log_3}{\log_3} = \frac{\log_3 20}{\log_3}$$

$$2x-1 = \frac{\log_3 20}{\log_3}$$

$$x = \frac{\log_3 20 + 1}{2}$$

$$2x = \frac{\log_3 20}{\log_3} + 1$$



A calculator screenshot showing the calculation of the solution for x. The input is $\frac{\log_3(20) + 1}{2}$ and the result is 1.863416514.

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$$3. \log_7(5x + 3) = 3$$

$$7^{\log_7(5x+3)} = 7^3$$

$$5x + 3 = 343$$

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$$4. \log(3x + 1) - \log \frac{1}{4} = 2$$

$$\log \frac{3x+1}{\frac{1}{4}} = 2$$

$$\log (3x+1)(4) = 2$$

$$10 \log (12x+4) = 10 \cdot 2$$

$$12x+4 = 100$$

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$$5. \log_4(x - 1) + \log_4(3x - 1) = 2$$

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Lesson Quiz: Part II

- 6.** A single cell divides every 5 minutes. How long will it take for one cell to become more than 10,000 cells?
- 7.** Use a table and graph to solve the equation $2^{3x} = 3^{3x-1}$.