7-6 The Natural Base, *e*

Warm Up
Lesson Presentation
Lesson Quiz

Warm Up

Simplify.

1. $\log 10^x$ $\quad x$
2. $\log_b b^{3w}$ $\quad 3w$
3. $10^{\log z}$ $\quad z$
4. $b^{\log_b (x-1)}$ $\quad x - 1$
5. $\left(\frac{1}{3}\right)3^{(x-1)}$ $\quad 3^{x-2}$

22. $\left(\frac{1}{4}\right)^x = 8^{x-1}$

$(2^{-2})^x = (2^3)^{x-1}$

$2^{-2x} = 2^{3x-3}$

$-2x = 3x - 3$

$-5x = -3$

$x = 0.6$

24. $\left(\frac{1}{2}\right)^{-x} = 1.6$

$\log\left(\frac{1}{2}\right)^{-x} = \log 1.6$

$-x \log \frac{1}{2} = \log 1.6$

$x = -\frac{\log 1.6}{\log \frac{1}{2}}$

$x \approx 0.678$

26. $3^{\frac{x}{2}+1} = 12.2$

$\log 3^{\frac{x}{2}+1} = \log 12.2$

$\left(\frac{x}{2} + 1\right)\log 3 = \log 12.2$

$\frac{x}{2} + 1 = \frac{\log 12.2}{\log 3}$

$x = 2\left(\frac{\log 12.2}{\log 3}\right)$

$x \approx 2.554$

28. $\log_3(7x) = \log_3(2x + 0.5)$

$7x = 2x + 0.5$

$5x = 0.5$

$x = 0.1$

30. $\log 5x - \log(15.5) = 2$

$\log \frac{5x}{15.5} = 2$

$\frac{x}{3.1} = 10^2$

$x = 310$

32. $\log x - \log\left(\frac{x}{100}\right) = x$

$\log\left(\frac{x}{\frac{x}{100}}\right) = x$

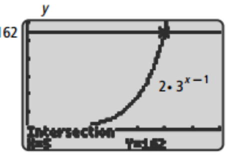
$\log 100 = x$

$2 = x$

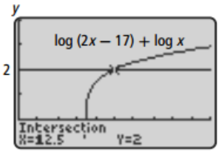
$\frac{x}{x/100}$

$x\left(\frac{100}{x}\right)$

34. $x = 5$



36. $x \geq 12.5$



46a. Decreasing; 0.987 is less than 1.

b. $t = 1980 - 1980 = 0$

$N(0) = 119(0.987)^0 = 119(1) = 119$

$t = 2000 - 1980 = 20$

$N(20) = 119(0.987)^{20} \approx 92$

There are 119,000 farms in 1980 and 92,000 in 2000.

c. $80000 = 119(0.987)^t$

$672.27 \approx (0.987)^t$

$30 \approx t$

$1980 + 30 = 2010$

The number of farms will be about 80,000 in 20

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Objectives

Use the number *e* to write and graph exponential functions representing real-world situations.

Solve equations and problems involving *e* or natural logarithms.

Vocabulary

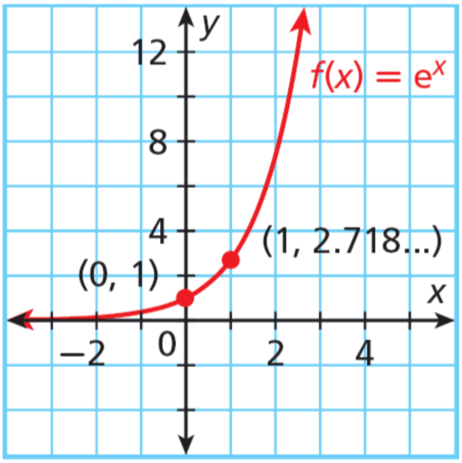
natural logarithm
natural logarithmic function

Recall the *compound interest formula* $A = P(1 + \frac{r}{n})^{nt}$, where A is the amount, P is the principal, r is the annual interest, n is the number of times the interest is compounded per year and t is the time in years.

Suppose that \$1 is invested at 100% interest ($r = 1$) compounded n times for one year as represented by the function $f(n) = P(1 + \frac{1}{n})^n$.

Exponential functions with e as a base have the same properties as the functions you have studied. The graph of $f(x) = e^x$ is like other graphs of exponential functions, such as $f(x) = 3^x$.

The domain of $f(x) = e^x$ is all real numbers. The range is $\{y | y > 0\}$.



D: ℝ

R: y > 0

Caution

The decimal value of e looks like it repeats: 2.718281828... The value is actually 2.71828182890... There is no repeating portion.

Example 1: Graphing Exponential Functions

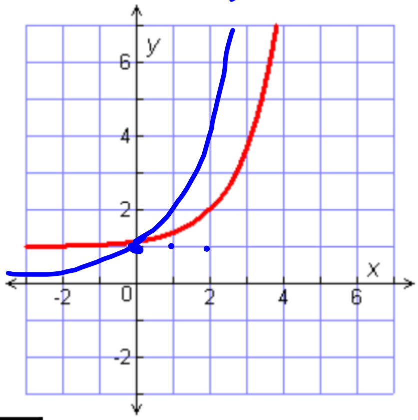
(0, 1)

Graph $f(x) = e^{(x-2)} + 1$. 1 unit

2

Make a table. Because *e* is irrational, the table values are rounded to the nearest tenth.

$(x-h)^2 + k \quad (x-2)^2 + 1$

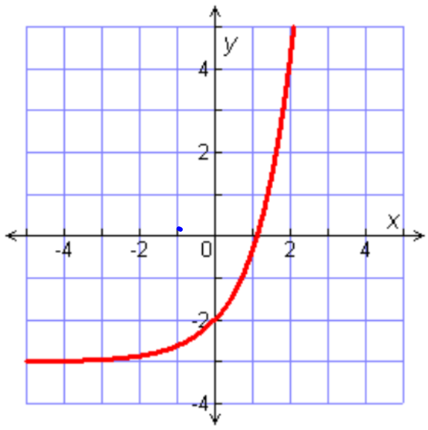


x	–2	–1	0	1	2	3	4
$f(x) = e^{x-2} + 1$	1.0	1.0	1.1	1.4	2	3.7	8.4

Check It Out! Example 1

Graph $f(x) = e^x - 3$.

Make a table. Because *e* is irrational, the table values are rounded to the nearest tenth.

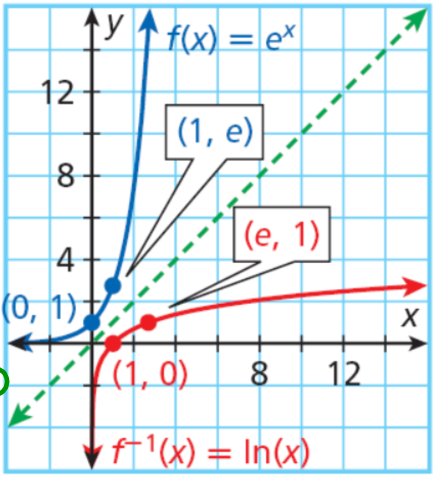


A logarithm with a base of e is called a **natural logarithm** and is abbreviated as "ln" (rather than as $\log_e$). Natural logarithms have the same properties as log base 10 and logarithms with other bases.

The **natural logarithmic function** $f(x) = \ln x$ is the inverse of the natural exponential function $f(x) = e^x$.

e^x D: R R: y>0

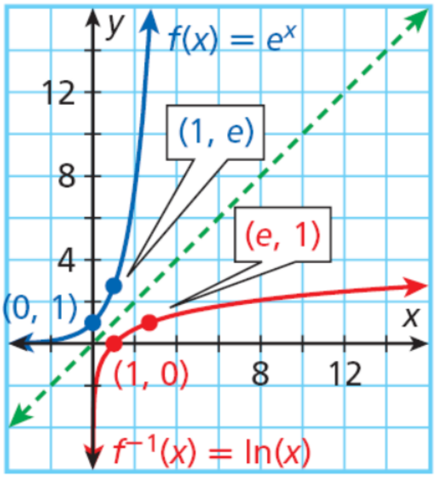
ln x D: x > 0 R: ℝ



The domain of $f(x) = \ln x$ is $\{x | x > 0\}$.

The range of $f(x) = \ln x$ is all real numbers.

All of the properties of logarithms from Lesson 7-4 also apply to natural logarithms.



Example 2: Simplifying Expression with *e* or ln

Simplify. $\log_e e$

$3\ln(x+1)$

A. $\cancel{\ln}\ \cancel{e}^{0.15t}$

$\ln e^{0.15t} = 0.15t$

B. $e^{3\ln(x+1)}$ $\cancel{e}^{\ln(x+1)^3}$

$e^{3\ln(x+1)} = (x+1)^3$

C. $\ln e^{2x} + \ln e^{x}$

$\ln e^{2x} + \ln e^{x} = 2x + x = 3x$

Check It Out! Example 2

Simplify.

a. $\ln e^{3.2}$

$\ln e^{3.2} = 3.2$

b. $e^{2\ln x}$ $2\ln x$

$e^{\ln x^2}$

$e^{2\ln x} = x^2$

c. $\ln e^{x+4y}$

$\ln e^{x+4y} = x + 4y$

The formula for continuously compounded interest is $A = Pe^{rt}$, where A is the total amount, P is the principal, r is the annual interest rate, and t is the time in years.

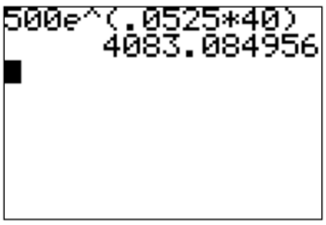
Example 3: Economics Application

What is the total amount for an investment of $500 invested at 5.25% for 40 years and compounded continuously?

$A = Pe^{rt}$

$A = 500e^{0.0525(40)}$ *Substitute 500 for P, 0.0525 for r, and 40 for t.*

$A \approx 4083.08$ *Use the e^x key on a calculator.*



The total amount is $4083.08.

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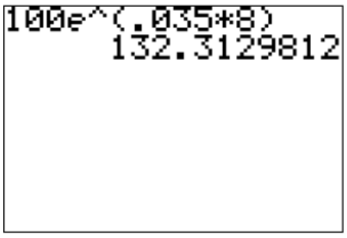
Check It Out! Example 3

What is the total amount for an investment of $100 invested at 3.5% for 8 years and compounded continuously?

$A = Pe^{rt}$

$A = 100e^{0.035(8)}$ *Substitute 100 for P, 0.035 for r, and 8 for t.*

$A \approx 132.31$ *Use the e^x key on a calculator.*



The total amount is $132.31.

The *half-life* of a substance is the time it takes for half of the substance to breakdown or convert to another substance during the process of decay. Natural decay is modeled by the function below.

N_0 is the initial amount (at $t = 0$). k is the decay constant.

$$N(t) = N_0 e^{-kt}$$

$N(t)$ is the amount remaining. t is the time.

Example 4: Science Application

Pluonium-239 (Pu-239) has a half-life of 24,110 years. How long does it take for a 1 g sample of Pu-239 to decay to 0.1 g? r= A=.1 P=1 t=?

Step 2 Write the decay function and solve for t.

$N(t) = N_0e^{-kt}$ $2^{\square} = \frac{1}{2}$

$\frac{1}{2} = 1e^{-k(24,110)}$

$\ln \frac{1}{2} = \ln e^{-24,110k}$

$\ln 2^{-1} = -24,110k$

$-\ln 2 = -24,110k$

$k = \frac{\ln 2}{24,110} \approx 0.000029$

$N(t) = N_0e^{-0.000029t}$

$0.1 = 1e^{-0.000029t}$

$\ln 0.1 = \ln e^{-0.000029t}$

$\ln 0.1 = -0.000029t$

$t = -\frac{\ln 0.1}{0.000029} \approx 80,000$

Check It Out! Example 4

Determine how long it will take for $\boxed{650}$ mg of a sample of chromium-51 which has a half-life of about 28 days to decay to 200 mg.

$N(t) = N_0e^{-kt}$

$\ln \frac{1}{2} = \ln e^{-k28}$

$\frac{\ln \frac{1}{2}}{-28} = \frac{-28k}{-28}$

$k = .0248$

$\frac{200}{650} = \frac{650e^{-.0248t}}{650}$

$\ln .3077 = \ln e^{-.0248t}$

$-1.179 = -.0248t$

$48 = t$

Lesson Quiz

Simplify.

1. $\ln e^{-10t}$ $-10t$

2. $e^{0.25 \ln t}$ $t^{0.25}$

3. $-\ln e^{x}$ $-x$

4. $2\ln e^{x^2}$ $2x^2$

5. What is the total amount for an investment of \$1000 invested at 7.25% for 15 years and compounded continuously? ≈ \$2966.85

6. The half-life of carbon-14 is 5730 years. What is the age of a fossil that only has 8% of its original carbon-14? ≈ 21,000 yr