7-7 Transforming Exponential and Logarithmic Functions

Warm Up

How does each function compare to its parent function?

1. $f(x) = 2(x - 3)^2 - 4$

vertically stretched by a factor of 2, translated 3 units right, translated 4 units down

2. $g(x) = (-x)^3 + 1$

reflected across the *y*-axis, translated 1 unit up

12. **Physics** Technetium-99m, a radioisotope used to image the skeleton and the heart muscle, has a half-life of about 6 hours. Find the decay constant. Use the decay function $\boxed{N(t) = N_0 e^{-kt}}$ to determine the amount of a 250 mg dose that remains after 24 hours.
$\approx$ 16 mg

$$f(24) = 250e^{-\left(\frac{\ln\frac{1}{2}}{-6}\right)(24)}$$

$$\frac{1}{2} = 1e^{-r6}$$

$$\ln\frac{1}{2} = \ln e^{-6r}$$

$$\ln\frac{1}{2} = -6r$$

$$\frac{\ln\frac{1}{2}}{-6} = .1155$$

Dec 4-8:22 AM

6. $\ln e^1$ **1**

7. x - y

8. $\ln e^{\left(-\frac{x}{3}\right)}$ $-\frac{x}{3}$

9. 2x

10. $e^{3\ln x}$ x^3

11. $ 9465.87

21. ≈ $5553.55

31. $x = \frac{e}{5} \approx 0.54$

33. $x = \pm\frac{e^5}{\sqrt{10}} \approx \pm 47$

35. $\{x \mid x > 0\}$

Dec 4-8:25 AM

Objectives

Transform exponential and logarithmic functions by changing parameters.

Describe the effects of changes in the coefficients of exponents and logarithmic functions.

You can perform the same transformations on exponential functions that you performed on polynomials, quadratics, and linear functions.

$y=2^x$ $y=3^x$ p. 537

Transformations of Exponential Functions

Transformation	$f(x)$ Notation	Examples	
Vertical translation	$f(x)+k$	$y=2^x+3$	3 units up
		$y=2^x-6$	6 units down
Horizontal translation	$f(x-h)$	$y=2^{x-2}$	2 units right
		$y=2^{x+1}$	1 unit left
Vertical stretch or compression	$af(x)$	$y=6(2^x)$	stretch by 6
		$y=\frac{1}{2}(2^x)$	compression by $\frac{1}{2}$
Horizontal stretch or compression ✓	$f\left(\frac{1}{b}x\right)$	$y=2^{\left(\frac{1}{5}x\right)}$	stretch by 5
		$y=2^{3x}$	compression by $\frac{1}{3}$
Reflection	$-f(x)$	$y=-2^x$	across x-axis
	$f(-x)$	$y=2^{-x}$	across y-axis

Helpful Hint

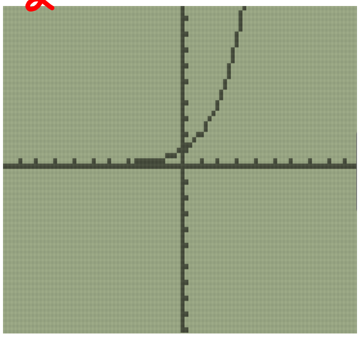
It may help you remember the direction of the shift if you think of "*h* is for horizontal."

Example 1: Translating Exponential Functions

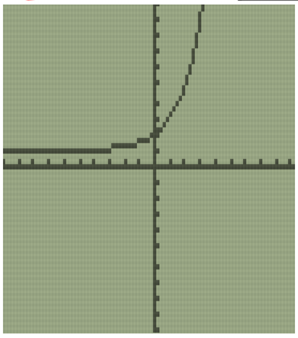
Make a table of values, and graph $g(x) = 2^{-x} + 1$. Describe the asymptote. Tell how the graph is transformed from the graph of the function $f(x) = 2^x$.

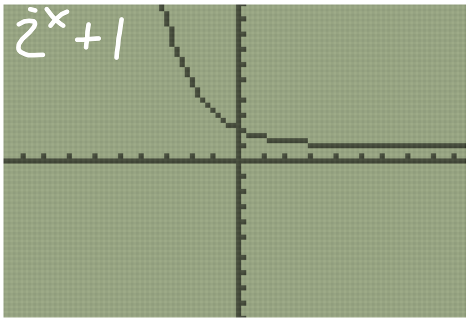
x	–3	–2	–1	0	1	2
$g(x)$	9	5	3	2	1.5	1.25

2^x



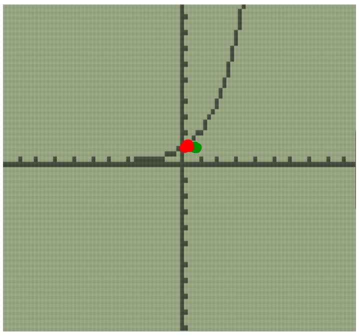
$2^x + 1$

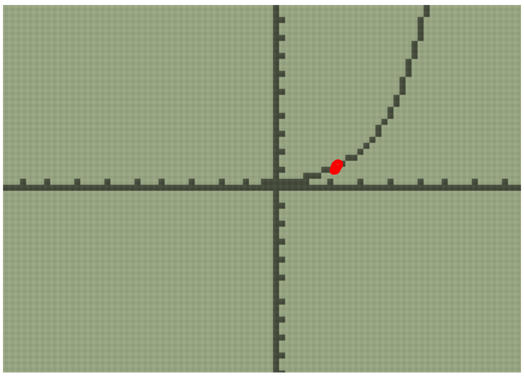




The asymptote is $y = 1$, and the graph approaches this line as the value of x increases. The transformation reflects the graph across the y-axis and moves the graph 1 unit up.

Make a table of values, and graph $f(x) = 2^{x-2}$. Describe the asymptote. Tell how the graph is transformed from the graph of the function $f(x) = 2^x$.



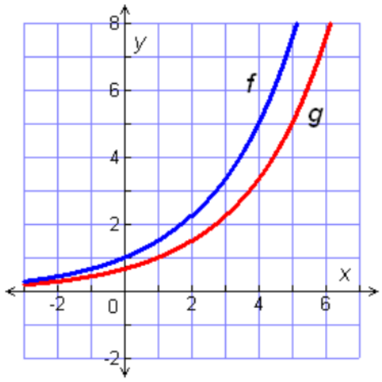


The asymptote is $y = 0$, and the graph approaches this line as the value of x decreases. The transformation moves the graph 2 units right.

Example 2: Stretching, Compressing, and Reflecting Exponential Functions

Graph the function. Find *y*-intercept and the asymptote. Describe how the graph is transformed from the graph of its parent function.

(0, 1)



A. $g(x) = \frac{2}{3}(1.5^x)$

parent function: $f(x) = 1.5^x$

y-intercept: $\frac{2}{3}$

asymptote: $y = 0$

The graph of $g(x)$ is a vertical compression of the parent function $f(x)$ 1.5^x by a factor of $\frac{2}{3}$.

Example 2: Stretching, Compressing, and Reflecting Exponential Functions

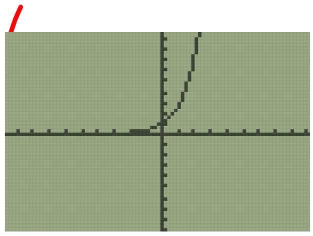
B. $h(x) = e^{-x+1}$ $-x+1=0$ $x=1$

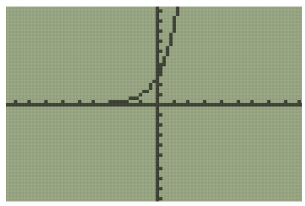
parent function: $f(x) = e^x$

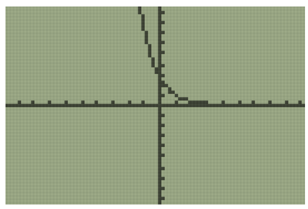
y-intercept: e

asymptote: $y = 0$

The graph of $h(x)$ is a reflection of the parent function $f(x) = e^x$ across the y-axis and a shift of 1 unit to the right. The range is $\{y|y > 0\}$.



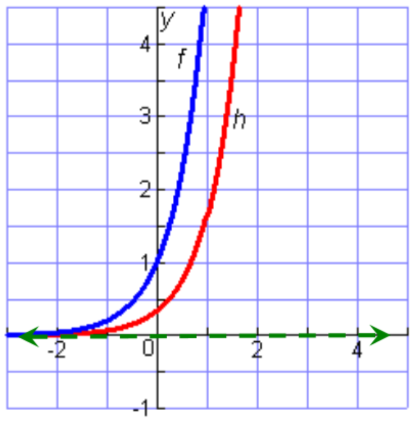




Check It Out! Example 2a

Graph the exponential function. Find *y*-intercept and the asymptote. Describe how the graph is transformed from the graph of its parent function.

$$h(x) = \frac{1}{3}(5^x)$$



parent function: $f(x) = 5^x$

y-intercept $\frac{1}{3}$

asymptote: 0

The graph of $h(x)$ is a vertical compression of the parent function $f(x) = 5^x$ by a factor of $\frac{1}{3}$.

Because a log is an exponent, transformations of logarithm functions are similar to transformations of exponential functions. You can stretch, reflect, and translate the graph of the parent logarithmic function $f(x) = \log_b x$.

Remember!

Transformations of ln x work the same way because ln$x$ means $\log_e x$.

Examples are given in the table below for $f(x) = \log x$.

ln p. 538 $y = \log_2 x + 3$

Transformation	$f(x)$ Notation	Examples	
Vertical translation	$f(x) + k$	$y = \log x + 3$	3 units up
		$y = \log x - 4$	4 units down
Horizontal translation	$f(x - h)$	$y = \log(x - 2)$	2 units right
		$y = \log(x + 1)$	1 unit left
Vertical stretch or compression	$af(x)$	$y = 6\log x$	stretch by 6
		$y = \frac{1}{2}\log x$	compression by $\frac{1}{2}$
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$	$y = \log\left(\frac{1}{5}x\right)$	stretch by 5
		$y = \log(3x)$	compression by $\frac{1}{3}$
Reflection	$-f(x)$	$y = -\log x$	across x-axis
	$f(-x)$	$y = \log(-x)$	across y-axis

$\log_2$

x^2

Example 3A: Transforming Logarithmic Functions

Graph each logarithmic function. Find the asymptote. Describe how the graph is transformed from the graph of its parent function.

$f(x)$ $\log x$

$$\boldsymbol{g(x) = 5\log x - 2}$$

Vertical stretch by a factor of 5

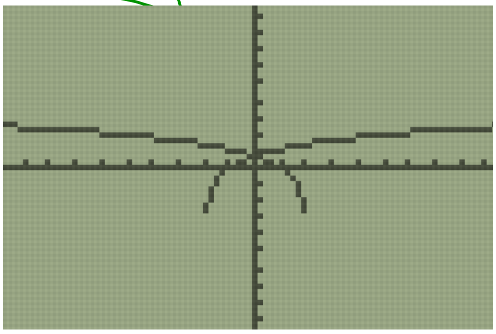
↓ 2 units

The graph of $g(x)$ is a vertical stretch of the parent function $f(x) = \log x$ by a factor of 5 and a translation 2 units down. asymptote: $x = 0$

Graph each logarithmic function. Find the asymptote. Describe how the graph is transformed from the graph of its parent function.

$-x+2=0$
$-x=-2$
$\boxed{x=2}$

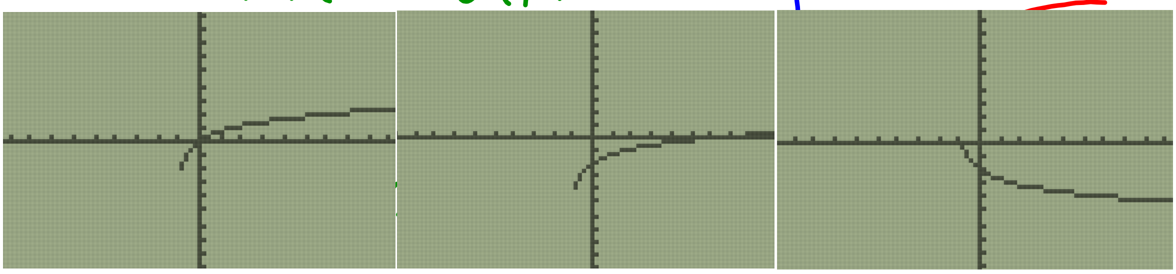
$h(x) = \ln(-x + 2)$



The graph of $h(x)$ is a reflection of the parent function $f(x)=\ln x$ across the y-axis and a shift of 2 units to the right.
D:$\{x|x < 2\}$

Graph the logarithmic $p(x) = -\ln(x + 1) - 2$. Find the asymptote. Then describe how the graph is transformed from the graph of its parent function.

horz. left 1 unit



The graph of $p(x)$ is a reflection of the parent function $f(x) = \ln x$ across the x-axis 1 unit left and a shift of 2 units down.

Example 4A: Writing Transformed Functions

Write each transformed function.

***f*(*x*) = 4^x is reflected across both axes and moved 2 units down.**

4^x	*Begin with the parent function.*
4^{-x}	*To reflect across the y-axis, replace x with –x.*
$-(4^{-x})$	*To reflect across the x-axis, multiply the function by –1.*
$g(x) = -(4^{-x}) - 2$	*To translate 2 units down, subtract 2 from the function.*

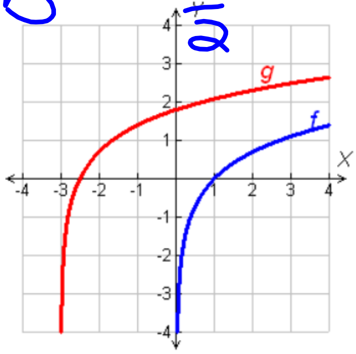
Example 4B: Writing Transformed Functions

$\boxed{f(x) = \ln x}$ **is compressed horizontally by a factor** $\boxed{\text{of } \frac{1}{2}}$ **and moved 3 units** $\boxed{\text{left.}}$

1 × 2

$\frac{1}{b} = \frac{1}{\frac{1}{2}} = 2$

$g(x) = \ln 2(x + 3)$



Ln x

ln 2(x+3)

Holt McDougal Algebra 2
Copyright © by Holt Mc Dougal. All Rights Reserved.

GUIDED PRACTICE

SEE EXAMPLE 1 p. 537

Make a table of values, and graph each function. Describe the asymptote. Tell how the graph is transformed from the graph of $f(x) = 3^x$.

1. $g(x) = 3^x + 2$
2. $h(x) = 3^x - 2$
3. $j(x) = 3^{x+1}$

SEE EXAMPLE 2 p. 538

Graph each exponential function. Find the y-intercept and the asymptote. Describe how the graph is transformed from the graph of its parent function.

4. $g(x) = 3(4^x)$
5. $h(x) = \frac{1}{3}(4^x)$
6. $j(x) = -\frac{1}{3}(4^x)$
7. $k(x) = -2(4^x)$
8. $m(x) = -(4^{-x})$
9. $n(x) = e^{2x}$

SEE EXAMPLE 3 p. 539

Graph each logarithmic function. Find the asymptote. Then describe how the graph is transformed from the graph of its parent function.

10. $g(x) = 2.5\log x$
11. $h(x) = 2.5\log(x + 3)$
12. $j(x) = -\frac{1}{3}\ln x + 1.5$

SEE EXAMPLE 4 p. 539

Write each transformed function by using the given parent function and the indicated transformations.

13. The parent exponential function $f(x) = 0.7^x$ is horizontally stretched by a factor of 3, reflected across the x-axis, and translated 2 units left.
14. The parent logarithmic function $f(x) = \log x$ is translated 12 units right, vertically compressed by a factor of $\frac{1}{2}$, and translated 25 units up.

SEE EXAMPLE 5 p. 540

15. **Forestry** The height of a poplar tree in feet, at age t years can be modeled by the function $h(t) = 6 + 3\ln(t + 1)$. Describe how the model is transformed from its parent function. Then use the model to predict the number of years when the height will exceed 17 feet.

Dec 4-10:41 AM

Check It Out! Example 4

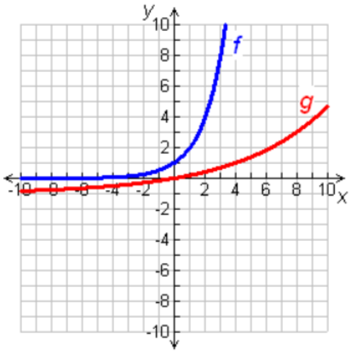
Write the transformed function when $f(x) = \log x$ is translated 3 units left and stretched vertically by a factor of 2.

$2f(x+3)$

$2\log(x+3)$

Lesson Quiz: Part I

1. Graph $g(x) = 2^{0.25x} - 1$. Find the asymptote. Describe how the graph is transformed from the graph of its parent function.



Lesson Quiz: Part II

2. Write the transformed function: $f(x) = \ln x$ is stretched by a factor of 3, reflected across the x-axis, and shifted by 2 units left.

Dec 4-7:43 AM