Date $\qquad$
Dear Family,
In Chapter 13 your child will begin to learn about trigonometry. The word trigonometry comes from Greek words meaning "triangle measurement."

A trigonometric function is defined by a rule that compares the lengths of two sides of a right triangle. The fundamental trigonometric functions are sine, cosine, and tangent. Three additional functions-cosecant, secant, and cotangent are defined as reciprocals. In trigonometry, $\theta$, is traditionally used to represent an angle measure.

| Trigonometric Functions |  |  |
| :--- | :--- | :--- |
| Definition | Symbols | Example |
| The sine of angle $\theta$ is the <br> ratio of the length of the <br> opposite leg to the length <br> of the hypotenuse. | $\sin \theta=\frac{\text { opp. }}{\text { hyp. }}$ | $\sin 60^{\circ}=\frac{4 \sqrt{3}}{8}=\frac{\sqrt{3}}{2}$ |
| The cosine of angle $\theta$ is <br> the ratio of the length <br> of the adjacent leg to the <br> length of the hypotenuse. | $\cos \theta=\frac{\text { adj. }}{\text { hyp. }}$ | $\cos 60^{\circ}=\frac{4}{8}=\frac{1}{2}$ |
| The tangent of angle $\theta$ is <br> the ratio of the length of <br> the opposite leg to the <br> length of the adjacent leg. | $\tan \theta=\frac{\text { opp. }}{\text { adj }}$ | $\tan 60^{\circ}=\frac{4 \sqrt{3}}{4}=\sqrt{3}$ |
| The cosecant of angle $\theta$ <br> is the reciprocal of the <br> sine function. | $\cos \theta=\frac{1}{\sin \theta}=\frac{\text { hyp. }}{\text { opp. }}$ | $\csc 60^{\circ}=\frac{8}{4 \sqrt{3}}=\frac{2}{\sqrt{3}}=\frac{2 \sqrt{3}}{3}$ |
| The secant of angle $\theta$ is <br> the reciprocal of the <br> cosine function. | $\sec \theta=\frac{1}{\cos \theta}=\frac{\text { hyp. }}{\text { adj. }}$ | $\sec 60^{\circ}=\frac{8}{4}=2$ |
| The cotangent of angle $\theta$ <br> is the reciprocal of the <br> tangent function. | $\cot \theta=\frac{1}{\tan \theta}=\frac{\text { adj. }}{\text { opp. }}$ | $\cos 60^{\circ}=\frac{4}{4 \sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ |

In a right triangle, the non-right angles must be acute, or $0^{\circ}<\theta<90^{\circ}$. However, trigonometric functions can be evaluated for any real value of $\theta$. Any positive or negative angle measure can be thought of as an angle of rotation, with the initial side on the positive $x$-axis and the terminal side rotated either counterclockwise (positive) or clockwise (negative). Angles beyond $360^{\circ}$ and $-360^{\circ}$ can also be created with multiple revolutions.


Most people are familiar with measuring angles with degrees. Another unit of angle measure is the radian, which is based on arc length. A complete circle has circumference $2 \pi r$ and $360^{\circ}$, so radians and degrees are related by
the ratio $\frac{2 \pi \text { radians }}{360^{\circ}}$, or $\frac{\pi \text { radians }}{180^{\circ}}$.
A unit circle is a circle centered at the origin of a coordinate grid with radius 1 unit. The unit circle at right shows several special angles measured in degrees and radians, and the corresponding $x$ - and $y$-coordinates of points on the unit circle. You may notice that there is a lot of symmetry between the quadrants, which will help your child memorize the unit circle.


The trigonometric functions use angle measures as inputs and output ratios. You can also use inverse trigonometric functions to work in reverse.
The inverse functions use ratios as inputs and output angle measures.
However, to truly be functions, the inverses have restricted domains. (The capital letters indicate that the functions have restricted domains.)

| Inverse Trigonometric Functions |  |  |
| :---: | :---: | :---: |
| Definition | Domain | Range |
| The inverse sine function is $\operatorname{Sin}^{-1} a=\theta$, where $\sin \theta=a$. | $\{a \mid-1 \leq a \leq 1\}$ | $\left\{\begin{array}{c} \left\{\theta \left\lvert\,-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right.\right\} \\ \left\{\theta \mid-90^{\circ} \leq \theta \leq 90^{\circ}\right\} \end{array}\right.$ |
| The inverse cosine function is $\operatorname{Cos}^{-1} a=\theta$, where $\cos \theta=a$. | $\{a \mid-1 \leq a \leq 1\}$ | $\begin{aligned} & \{\theta \mid 0 \leq \theta \leq \pi\} \\ & \left\{\theta \mid 0^{\circ} \leq \theta \leq 180^{\circ}\right\} \end{aligned}$ |
| The inverse tangent function is $\operatorname{Tan}^{-1} a=\theta$, where $\tan \theta=a$. | $\{a \mid-\infty<a<\infty\}$ | $\left\{\begin{array}{l} \left\{\theta \left\lvert\,-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right.\right\} \\ \left\{\theta \mid-90^{\circ}<\theta<90^{\circ}\right\} \end{array}\right.$ |

The chapter concludes with two important laws that can be used to "solve" a triangle, which means to find all of the angle measures and side lengths when only a few are known.

Law of Sines:

$$
\begin{aligned}
& \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A \\
& b^{2}=a^{2}+c^{2}-2 a c \cos B \\
& c^{2}=a^{2}+b^{2}-2 a b \cos C
\end{aligned}
$$

Law of Cosines: $\quad a^{2}=b^{2}+c^{2}-2 b c \cos A$


You child will continue to study trigonometry in Chapter 14. For additional resources, visit go.hrw.com and enter the keyword MB7 Parent.

