LESSON Reteach

Solving Linear Systems in Three Variables

You know how to solve a system of two linear equations in two variables using the **elimination method**. The same method can be used to solve a system of three linear equations in three variables.

$$\begin{cases} x - y + 2z = 8 \\ 2x + y - z = -2 \\ x + 2y + z = 2 \end{cases}$$

The first and second equations have opposite coefficients of *y*. So adding these two equations will eliminate *y*.

$$x - y + 2z = 8$$
$$+2x + y - z = -2$$
$$3x + z = 6$$

Multiply the first equation by 2 and add to the third equation to eliminate y.

$$2x - 2y + 4z = 16$$
$$+ x + 2y + z = 2$$
$$3x + 5z = 18$$

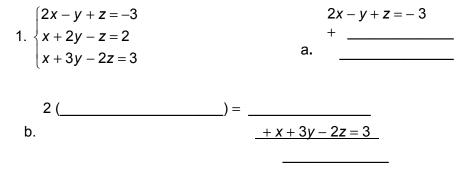
Now you have two equations in two variables. Solve using the elimination method for a system of two equations.

$$\begin{cases} 3x + z = 6 \\ 3x + 5z = 18 \end{cases}$$

Solving this system gives x = 1 and z = 3. Substituting these values in any of the original equations gives y = -1.

So the solution is the ordered triple (1, -1, 3)

Show the steps you would use to eliminate the variable z.



c. Give the resulting system of two equations.

Reteach LESSON 3-6

Solving Linear Systems in Three Variables (continued)

Linear systems in three variables are classified by their solutions.

Exactly One Solution Independent	Infinitely Many Solutions Dependent	No Solution Inconsistent
Three planes intersect at one point.	Three planes intersect at a line.	All three planes never intersect.
nuosiny. $n = y = z = z$	Id the second and third uations to eliminate <i>y</i> .	x + y + z = 2 + $\frac{x - y + z = 1}{2x + 2z = 3}$
		-2x - 2z = -2 + $2x + 2z = 3$ 0 = 1 X
Since 0 does not equal 1, the s	ystem has no solution and is in	consistent.
$\int x + 2y + 4z = 3$		x + 2y + 4z = 3
Classify: $\begin{cases} 4x - 2y - 6z = 2\\ 2x - y - 3z = 1 \end{cases}$	Add the first and second equations.	$+\frac{4x-2y-6z=2}{5x}$
Multiply the third equation first equation.	n by 2. Add to the	= 4x - 2y - 6z = 2 + x + 2y + 4z = 3 5x - 2z = 5
low you have a system with tw	vo identical equations.	5x - 2z = 5
Subtracting the equations gives	s 0 = 0.	$ \begin{cases} 5x - 2z = 5 \\ 5x - 2z = 5 \end{cases} $
he system has infinitely many	solutions and is dependent.	

Classify each system and determine the number of solutions.

	$\int x + z = 0$		(y-z=0
2.	x + y + 2z = 3	3.	x - 3z = -1
	y + z = 2		$\left(-x+3y=1\right)$

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