Name ______________________ Date ____________ Class____________

LESSON 3-6

Reteach

Solving Linear Systems in Three Variables

You know how to solve a system of two linear equations in two variables using the **elimination method**. The same method can be used to solve a system of three linear equations in three variables.

$$\begin{cases} x - y + 2z = 8 \\ 2x + y - z = -2 \\ x + 2y + z = 2 \end{cases}$$

The first and second equations have opposite coefficients of *y*. So adding these two equations will eliminate *y*.

$$\begin{array}{r} x - y + 2z = 8 \\ \underline{+2x + y - z = -2} \\ 3x + z = 6 \end{array}$$

Multiply the first equation by 2 and add to the third equation to eliminate *y*.

$$\begin{array}{r} 2x - 2y + 4z = 16 \\ \underline{+\ x + 2y + z = 2\ } \\ 3x + 5z = 18 \end{array}$$

Now you have two equations in two variables. Solve using the elimination method for a system of two equations.

$$\begin{cases} 3x + z = 6 \\ 3x + 5z = 18 \end{cases}$$

Solving this system gives $x = 1$ and $z = 3$. Substituting these values in any of the original equations gives $y = -1$.

So the solution is the ordered triple (1, –1, 3)

Show the steps you would use to eliminate the variable *z*.

1. $\begin{cases} 2x - y + z = -3 \\ x + 2y - z = 2 \\ x + 3y - 2z = 3 \end{cases}$

a. $2x - y + z = -3$

$+$ ______________

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b. 2 (____________________) = ______________

$+\ x + 3y - 2z = 3$

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c. Give the resulting system of two equations. ______________________________

Name ______________________________ Date ______________ Class______________

LESSON 3-6

Reteach

Solving Linear Systems in Three Variables (continued)

Linear systems in three variables are classified by their solutions.

Exactly One Solution Independent	Infinitely Many Solutions Dependent	No Solution Inconsistent
Three planes intersect at one point.	Three planes intersect at a line.	All three planes never intersect.

Classify: $\begin{cases} x + z = 1 \\ x + y + z = 2 \\ x - y + z = 1 \end{cases}$

Add the second and third equations to eliminate *y*.

$$\begin{array}{r} x + y + z = 2 \\ +\underline{\;x - y + z = 1\;} \\ 2x + 2z = 3 \end{array}$$

Solve: $\begin{cases} x + z = 1 \\ 2x + 2z = 3 \end{cases}$

Multiply the first equation by −2. Then add.

$$\begin{array}{r} -2x - 2z = -2 \\ +\underline{2x + 2z = 3} \\ 0 = 1 \end{array} \quad \textbf{X}$$

Since 0 does not equal 1, the system has no solution and is inconsistent.

Classify: $\begin{cases} x + 2y + 4z = 3 \\ 4x - 2y - 6z = 2 \\ 2x - y - 3z = 1 \end{cases}$

Add the first and second equations.

$$\begin{array}{r} x + 2y + 4z = 3 \\ +\underline{\;4x - 2y - 6z = 2\;} \\ 5x \qquad -2z = 5 \end{array}$$

Multiply the third equation by 2. Add to the first equation.

$$\begin{array}{r} 4x - 2y - 6z = 2 \\ +\underline{\;x + 2y + 4z = 3\;} \\ 5x \qquad -2z = 5 \end{array}$$

Now you have a system with two identical equations.

Subtracting the equations gives 0 = 0.

The system has infinitely many solutions and is dependent.

$\begin{cases} 5x - 2z = 5 \\ 5x - 2z = 5 \end{cases}$

Classify each system and determine the number of solutions.

2. $\begin{cases} x + z = 0 \\ x + y + 2z = 3 \\ y + z = 2 \end{cases}$

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3. $\begin{cases} y - z = 0 \\ x - 3z = -1 \\ -x + 3y = 1 \end{cases}$

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