8-8 Solving Radical Equations and Inequalities

Warm Up

Simplify each expression. Assume all variables are positive.

1. $2\sqrt{27x} + 3\sqrt{12x}$ $12\sqrt{3x}$
2. $\sqrt{72y^5}$ $6y^2\sqrt{2y}$
3. $\sqrt[3]{(x+2)^3}$ $x+2$
4. $\sqrt{2(48y)}$ $4\sqrt{6y}$

Write each expression in radical form.

5. $(x+6)^{\frac{1}{2}}$ $\sqrt{x+6}$
6. $(3y+4)^{\frac{3}{5}}$ $\sqrt[5]{(3y+4)^3}$

$2\sqrt{27x} + 3\sqrt{12x}$

$2\sqrt{9}\sqrt{3x} + 3\sqrt{4}\sqrt{3x}$

$2 \cdot 3\sqrt{3x} + 3 \cdot 2\sqrt{3x}$

$6\sqrt{3x} + 6\sqrt{3x} = 12\sqrt{3x}$

$\sqrt{72y^5}$

$\sqrt{9}\sqrt{8y^5}$ y^{4+1}

$3\sqrt{4}\sqrt{2y^5}$

$3(2)\sqrt{2y^5}$

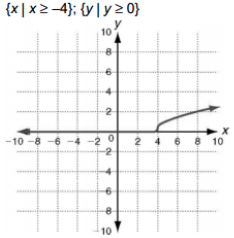
$\sqrt{2(48y)}$ $6\sqrt{y^4}\sqrt{y}$

$\sqrt{2}\sqrt{16}\sqrt{3y}$ $6y^2\sqrt{2y}$

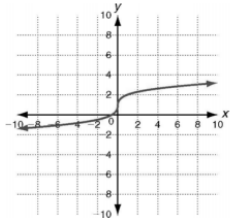
$4\sqrt{6y}$

Practice B

1. $\{x \mid x \geq -4\}$; $\{y \mid y \geq 0\}$



2. all real numbers; all real numbers



3. Vertical stretch by a factor of 4 and translate 8 units left
4. Reflection across the x-axis, horizontal compression by a factor of $\frac{1}{3}$, and translate 2 units up
5. $g(x) = 7\sqrt{-x} - 3$
6. $g(x) = -\sqrt{2(x-2)}$
7. 0.29 cm

8-8 Solving Radical Equations and Inequalities

Objective

Solve radical equations and inequalities.

Vocabulary

radical equation
radical inequality

8-8 Solving Radical Equations and Inequalities

A **radical equation** contains a variable within a radical. Recall that you can solve quadratic equations by taking the square root of both sides. Similarly, radical equations can be solved by raising both sides to a power.

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Solving Radical Equations

Steps	Example
1. Isolate the radical.	$\sqrt[3]{x} - 2 = 0$ $\sqrt[3]{x} = 2$
2. Raise both sides of the equation to the power equal to the index of the radical.	$(\sqrt[3]{x})^3 = (2)^3$
3. Simplify and solve.	$x = 8$

Remember!

For a square root, the index of the radical is 2.

$\sqrt{x+1} = \sqrt[2]{x+1}$

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Example 1B: Solving Equations Containing One Radical

Solve each equation.

$7\sqrt[3]{5x-7} = 84$

$\frac{7\sqrt[3]{5x-7}}{7} = \frac{84}{7}$ *Divide by 7.*

$\sqrt[3]{5x-7} = 12$ *Simplify.*

$(\sqrt[3]{5x-7})^3 = (12)^3$ *Cube both sides.*

$5x - 7 = 1728$ *Simplify.*

$5x = 1735$ *Solve for x.*

$x = 347$

Check

$7\sqrt[3]{5x-7} = 84$

$7\sqrt[3]{5(347)-7}$ | 84

$7\sqrt[3]{1728}$ | 84

84 | 84 ✓

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Check It Out! Example 1a

Solve the equation.

$4+\sqrt{x-1}=5$

$\sqrt{x-1}=5-4$ *Subtract 4.*

$\sqrt{x-1}=1$ *Simplify.*

$\left(\sqrt{x-1}\right)^2=(1)^2$ *Square both sides.*

$x-1=1$ *Simplify.*

$x=2$ *Solve for x.*

Check

$4+\sqrt{x-1}=5$

$4+\sqrt{2-1}$	5
$4+\sqrt{1}$	5
5	5 ✓

Example 2: Solving Equations Containing Two Radicals

Solve $\sqrt{7x+2}=3\sqrt{3x-2}$

$y_1=\sqrt{7x+2}-3\sqrt{3x-2}$

$\left(\sqrt{7x+2}\right)^2=\left(3\sqrt{3x-2}\right)^2$

$7x+2=9(3x-2)$

$7x+2=27x-18$

$20=20x$

$1=x$

Check

$\sqrt{7x+2}=3\sqrt{3x-2}$

$\sqrt{7(1)+2}$	$3\sqrt{3(1)-2}$
3	3 ✓

Raising each side of an equation to an even power may introduce extraneous solutions.

Helpful Hint

You can use the intersect feature on a graphing calculator to find the point where the two curves intersect.

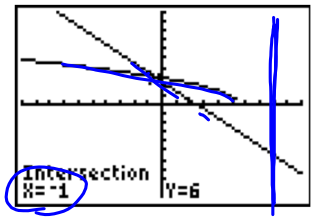
Example 3: Solving Equations with Extraneous Solutions

$-3x+33\geq 0$

Solve $\sqrt{-3x+33}=5-x$

Method 1 Use a graphing calculator.

Let **Y1** $=\sqrt{-3x+33}$ and **Y2** $=5-x$.



The graphs intersect in only one point, so there is exactly one solution.

The solution is $x=-1$

Example 3 Continued

Method 2 Use algebra to solve the equation.

Step 1 Solve for x.

$\sqrt{-3x+33} = 5-x$

$\left(\sqrt{-3x+33}\right)^2 = (5-x)^2$ F.O.I.L.

$-3x + 33 = 25 - 10x + x^2$

$0 = x^2 - 7x - 8$

$0 = (x-8)(x+1)$

$x - 8 = 0$ or $x + 1 = 0$

$x = 8$ or $x = -1$

$\sqrt{-3x+33} = 5-x$	
$\sqrt{-3(-1)+33}$	$5-(-1)$
6	6 ✓

$\sqrt{-3x+33} = 5-x$	
$\sqrt{-3(8)+33}$	$5-8$
3	-3 ✗

Because $x = 8$ is extraneous, the only solution is $x = -1$.

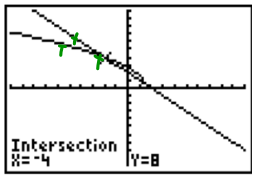
Check It Out! Example 3b

Solve each equation.

$\sqrt{-9x+28} = -x+4$

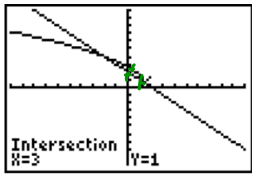
Method 1 Use a graphing calculator.

Let **Y1** $= \sqrt{-9x+28}$ and **Y2** $= -x+4$.



The graphs intersect in two points, so there are two solutions.

The solutions are $x = -4$ and $x = 3$.



Check It Out! Example 3b Continued

Method 2 Use algebra to solve the equation.

Step 1 Solve for x.

$\sqrt{-9x+28} = -x+4$

$\left(\sqrt{-9x+28}\right)^2 = (-x+4)^2$

$-9x + 28 = x^2 - 8x + 16$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x + 4 = 0$ or $x - 3 = 0$

$x = -4$ or $x = 3$

$\sqrt{-9x+28} = -x+4$	
$\sqrt{-9(-4)+28}$	$-(-4)+4$
8	8 ✓

$\sqrt{-9x+28} = -x+4$	
$\sqrt{-9(3)+28}$	$-(3)+4$
1	1 ✓

Remember!

To find a power, multiply the exponents.

$\left[(x+12)^{\frac{1}{2}}\right]^2$

$(x+12)^{\frac{1}{2}\cdot 2}$

$x + 12$

Check It Out! Example 4c

$3(x + 6)^{\frac{1}{2}} = 9$

$[3(x + 6)^{\frac{1}{2}}]^2 = (9)^2$

$9(x + 6) = 81$

$9x + 54 = 81$

$9x = 27$

$x = 3$

$\frac{3(x + 6)^{\frac{1}{2}}}{3} = \frac{9}{3}$

$(x+6)^{\frac{1}{2}} = 3$

$\sqrt{x+6}^2 = 3^2$

$x+6 = 9$

$x = 3$

A **radical inequality** is an inequality that contains a variable within a radical. You can solve radical inequalities by graphing or using algebra.

Remember!

A radical expression with an even index and a negative radicand has no real roots.

Example 5: Solving Radical Inequalities

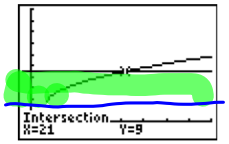
Solve $\sqrt{2x-6} + 3 \le 9$.

Method 1 Use a graph and a table.

On a graphing calculator, let **Y1** = $\sqrt{2x-6} + 3$ and **Y2** = 9. The graph of **Y1** is at or below the graph of **Y2** for values of x between 3 and 21. Notice that **Y1** is undefined when < 3.

X	Y1	Y2
18	8.4772	9
19	8.6568	9
20	8.831	9
21	9	9
22	9.1644	9
23	9.3246	9
24	9.4807	9

X=21



$x \le 21$

Example 5 Continued

Method 2 Use algebra to solve the inequality.

Step 1 Solve for x.

$\sqrt{2x-6} + 3 \le 9$

$\sqrt{2x-6} \le 6$

$\left(\sqrt{2x-6}\right)^2 \le (6)^2$

$2x \le 42$

$x \le 21$

Step 2 Consider the radicand.

$2x - 6 \ge 0$

$2x \ge 6$

$x \ge 3$

The solution $\sqrt{2x-6} + 3 \le 9$ is $x \ge 3$ and $x \le 21$, or $3 \le x \le 21$.

Example 6: Automobile Application

The time *t* in seconds that it takes a car to travel a quarter mile when starting from a full stop can be estimated by using the formula $t = 5.825\sqrt[3]{\frac{w}{P}}$, where *w* is the weight of the car in pounds and *P* is the power delivered by the engine in horsepower. If the quarter-mile time from a 3590 lb. car is 13.4 *s*, how much power does its engine deliver? Round to the nearest horsepower.

$$t = 5.825\sqrt[3]{\frac{W}{P}}$$

$$\left(\frac{13.4}{5.825}\right) = \frac{5.825\sqrt[3]{\frac{3590}{P}}}{5.825}$$

$$2.3^3 = \sqrt[3]{\frac{3590}{P}}^3$$

$$(2.3)^3 = \frac{3590}{P} \qquad \frac{(2.3)^3 P}{(2.3)^3} = \frac{3590}{(2.3)^3}$$

$$P = 295$$

$$\frac{3590}{\left(\frac{13.4}{5.825}\right)^3}$$

Example 6 Continued

Use the formula to determine the amount of horsepower the 3590 lb car has if it finishes the quarter-mile in 13.4*s*.

$$t = 5.825\sqrt[3]{\frac{W}{P}}$$

$$13.4 = 5.825\sqrt[3]{\frac{3590}{P}}$$ *Substitute 13.4 for t and 3590 for w.*

$$(13.4)^3 = \left(5.825\sqrt[3]{\frac{3590}{P}}\right)^3$$ *Cube both sides.*

$$2406.104 \approx 197.646\left(\frac{3590}{P}\right)$$ *Simplify.*

$$2406.104P \approx 709{,}548.747$$ *Solve for P.*

$$P \approx 295$$ The engine delivers a power of about 295 hp.

Lesson Quiz: Part I

Solve each equation or inequality.

1. $7 + \sqrt{2x+4} = 13$

2. $3\sqrt[3]{x+4} = \sqrt[3]{2x-17}$

3. $\sqrt{2x+12} = x+2$

4. $4(x-5)^{\frac{1}{2}} = 12$

5. $\sqrt[3]{x+5} \geq 4$

6. The radius *r* in feet of a spherical water tank can be determined by using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$, where *V* is the volume of the tank in cubic feet. To the nearest cubic foot, what is the volume of a spherical tank with a radius of 32 ft?

