9-1 Identifying Quadratic Functions

Warm Up

Lesson Presentation

Lesson Quiz

9-1 Identifying Quadratic Functions

Warm Up

1. Evaluate $x^2 + 5x$ for $x = 4$ and $x = -3$.

2. Generate ordered pairs for the function $y = x^2 + 2$ with the given domain.

D: {–2, –1, 0, 1, 2}

		–1	0	1	2
	6	3	2	3	6

9-1 Identifying Quadratic Functions

Objectives

Identify quadratic functions and determine whether they have a minimum or maximum.

Graph a quadratic function and give its domain and range.

9-1 Identifying Quadratic Functions

Vocabulary

quadratic function
parabola
vertex
minimum
maximum

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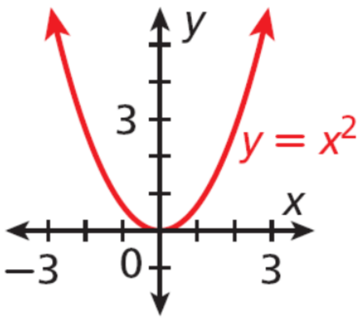
9-1 Identifying Quadratic Functions

The function $y = x^2$ is shown in the graph. This function is a *quadratic function*. A **quadratic function** is any function that can be written in the standard form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The function $y = x^2$ can be written as $y = 1x^2 + 0x + 0$, where $a = 1$, $b = 0$, and $c = 0$.



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9-1 Identifying Quadratic Functions

In Lesson 5-1, you identified linear functions by finding that a constant change in x corresponded to a constant change in y. The differences between y-values for a constant change in x-values are called *first differences*.

Constant change in x-values: +1, +1, +1, +1

x	0	1	2	3	4
$y = x^2$	0	1	4	9	16

First differences: +1, +3, +5, +7

Second differences: +2, +2, +2

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9-1 Identifying Quadratic Functions

Example 1A: Identifying Quadratic Functions

Tell whether the function is quadratic. Explain.

x	*y*
–2	–9
–1	–2
0	–1
1	0
2	7

Changes in x: +1, +1, +1, +1

First differences in y: +7, +1, +1, +7

Second differences: –6, +0, +6

Since you are given a table of ordered pairs with a constant change in x-values, see if the second differences are constant.

Find the first differences, then find the second differences.

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Example 1B: Identifying Quadratic Functions

Tell whether the function is quadratic. Explain.

$y = 7x + 3$

not

$y - 10x^2 = 9$

$+10x^2 \quad +10x^2$

$y = 10x^2 + 9$

$A = 10 \quad B = 0 \quad C = 9$

Helpful Hint

Only *a* cannot equal 0. It is okay for the values of *b* and *c* to be 0.

9-1 Identifying Quadratic Functions

Check It Out! Example 1b

Tell whether the function is quadratic. Explain.

$\mathbf{y + x = 2x^2}$

$$
\begin{aligned}
y + x &= 2x^2 \\
-x \quad & \quad -x \\
y &= 2x^2 - x
\end{aligned}
$$

Try to write the function in the form $y = ax^2 + bx + c$ by solving for y. Subtract x from both sides.

This is a quadratic function because it can be written in the form $y = ax^2 + bx + c$ where $a = 2$, $b = -1$, and $c = 0$.

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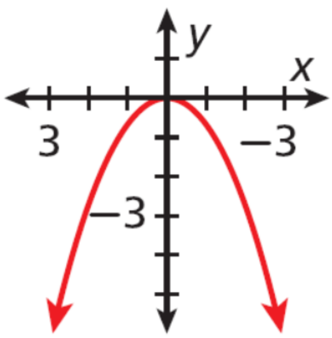
The graph of a quadratic function is a curve called a **parabola**. To graph a quadratic function, generate enough ordered pairs to see the shape of the parabola. Then connect the points with a smooth curve.



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Check It Out! Example 2b

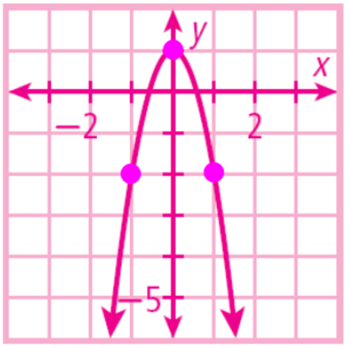
Use a table of values to graph the quadratic function.

$y = -3x^2 + 1$

x	y
–2	–11
–1	–2
0	1
1	–2
2	–11

Make a table of values. Choose values of x and use them to find values of y.

Graph the points. Then connect the points with a smooth curve.



9-1 Identifying Quadratic Functions

When a quadratic function is written in the form $y = ax^2 + bx + c$, the value of a determines the direction a parabola opens.

- A parabola opens upward when $a > 0$.
- A parabola opens downward when $a < 0$.

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Example 3A: Identifying the Direction of a Parabola

Tell whether the graph of the quadratic function opens upward or downward. Explain.

$$y - \frac{1}{3}x^2 = x - 3$$

$$+\frac{1}{3}x^2 \quad +\frac{1}{3}x^2$$

$$y = \frac{1}{3}x^2 + x - 3$$

$$y = \frac{1}{3}x^2 + x - 3$$

$$a = \frac{1}{3}$$

$$y = 5x - 3x^2$$

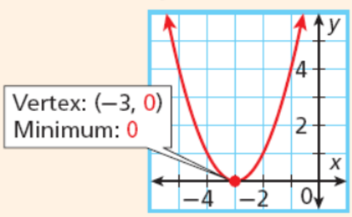
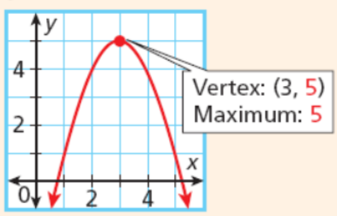
$$y = -3x^2 + 5x$$

$$a = -3$$

9-1 Identifying Quadratic Functions

The highest or lowest point on a parabola is the **vertex**. If a parabola opens upward, the vertex is the lowest point. If a parabola opens downward, the vertex is the highest point.

Minimum and Maximum Values

WORDS	If $a > 0$, the parabola opens upward, and the y-value of the vertex is the **minimum** value of the function.	If $a < 0$, the parabola opens downward, and the y-value of the vertex is the **maximum** value of the function.
GRAPHS	$y = x^2 + 6x + 9$ 	$y = -x^2 + 6x - 4$ 

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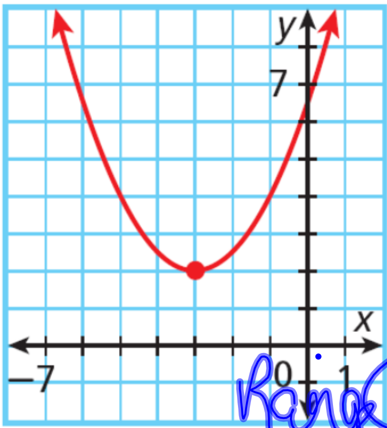
Example 4: Identifying the Vertex and the Minimum or Maximum

Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

A.



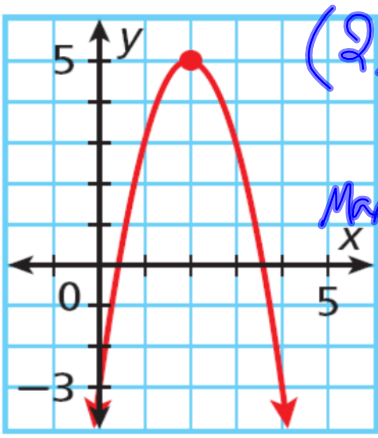
Vertex (-3,2)

Max/Min

Range: y ≥ 2

y=2

B.



(2,5)

Max=5

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Domain: *Numbers that can go into an equation.*

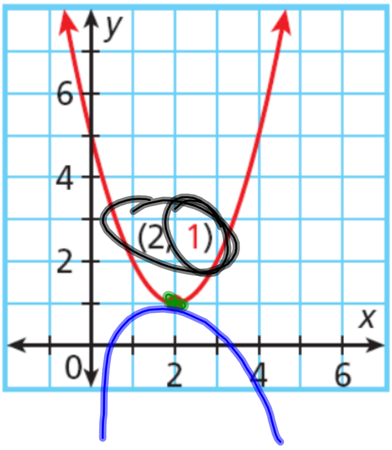
- *Looking a the graph how far left are right will it go*
- *Are the x* values of the equation.
- Input values

Range: *Numbers that can come out of the equation.*

- *Looking at the graph how far up or down will it go*
- *Are the y* values in the equation.
- Output values

9-1 Identifying Quadratic Functions

Unless a specific domain is given, you may assume that the domain of a quadratic function is all real numbers. You can find the range of a quadratic function by looking at its graph.



For the graph of $y = x^2 - 4x + 5$, the range begins at the minimum value of the function, where $y = 1$. All the y-values of the function are greater than or equal to 1. So the range is $y \geq 1$.

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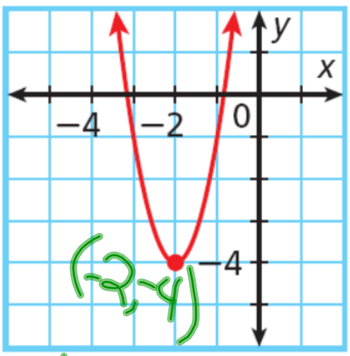
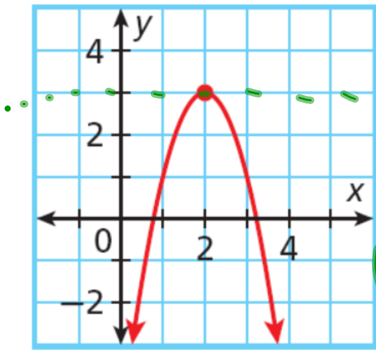
Check It Out! Example 5a

Find the domain and range.



MIN = -4

$y \geq -4$



(2,3)
MAX = 3
y = 3
R: $y \leq 3$

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x-intercept

Recall that an x-intercept of a function is a value of x when y = 0. A zero of a function is an x-value that makes the function equal to 0. So a zero of a function is the same as an x-intercept of a function. Since a graph intersects the x-axis at the point or points containing an x-intercept, these intersections are also at the zeros of the function. A quadratic function may have one, two, or no zeros.

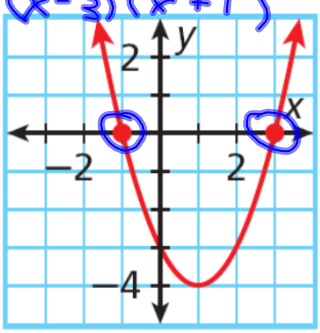
9-1 Identifying Quadratic Functions

Example 1A: Finding Zeros of Quadratic Functions From Graphs

Find the zeros of the quadratic function from its graph. Check your answer.

$y = x^2 - 2x - 3$

$(x-3)(x+1)$



Check

$y = x^2 - 2x - 3$

$y = (-1)^2 - 2(-1) - 3$
$= 1 + 2 - 3 = 0$ ✓

$y = 3^2 - 2(3) - 3$
$= 9 - 6 - 3 = 0$ ✓

The zeros appear to be –1 and 3.

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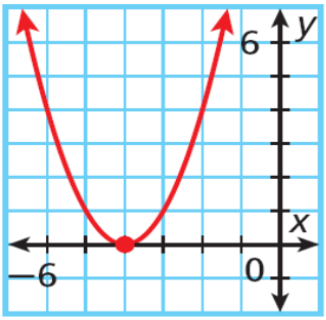
Example 1B: Finding Zeros of Quadratic Functions From Graphs

Find the zeros of the quadratic function from its graph. Check your answer.

$y = x^2 + 8x + 16$



The zero appears to be –4.

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Helpful Hint

Notice that if a parabola has only one zero, the zero is the x-coordinate of the vertex.

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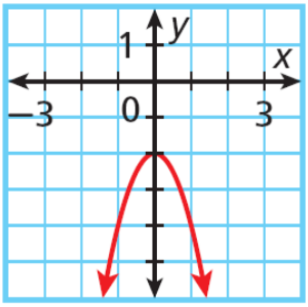
Example 1C: Finding Zeros of Quadratic Functions From Graphs

Find the zeros of the quadratic function from its graph. Check your answer.

$y = -2x^2 - 2$

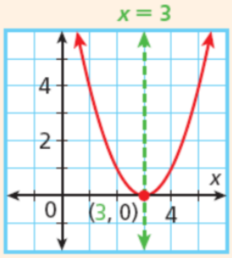
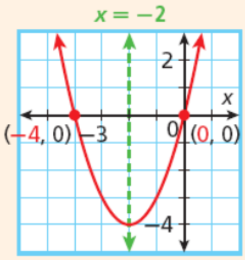


9-1 Identifying Quadratic Functions

A vertical line that divides a parabola into two symmetrical halves is the <u>*axis of symmetry*</u>*. The axis of symmetry always passes through the vertex of the parabola. You can use the zeros to find the axis of symmetry.*

9-1 Identifying Quadratic Functions

Finding the Axis of Symmetry by Using Zeros

WORDS	NUMBERS	GRAPH
One Zero		
If a function has one zero, use the *x*-coordinate of the vertex to find the axis of symmetry.	Vertex: (3, 0) Axis of symmetry: $x = 3$	
Two Zeros		
If a function has two zeros, use the average of the two zeros to find the axis of symmetry.	$\frac{-4+0}{2} = \frac{-4}{2} = -2$ Axis of symmetry: $x = -2$	

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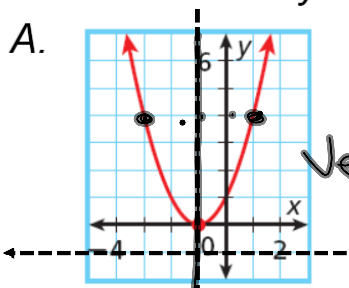
Example 2: Finding the Axis of Symmetry by Using Zeros

Find the axis of symmetry of each parabola.

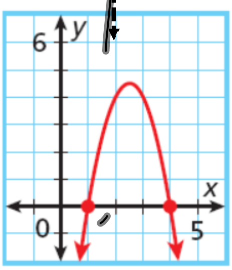
A.



$x = -1$

Vertex $(-1, 0)$

B.



$1 + 4 = 5 \quad \frac{5}{2} = 2.5$

9-1 Identifying Quadratic Functions

If a function has no zeros or they are difficult to identify from a graph, you can use a formula to find the axis of symmetry. The formula works for all quadratic functions.

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Finding the Axis of Symmetry by Using the Formula

FORMULA	EXAMPLE
For a quadratic function $y = ax^2 + bx + c$, the axis of symmetry is the vertical line $x = -\frac{b}{2a}$.	$y = 2x^2 + 4x + 5$ $x = -\frac{b}{2a}$ $= -\frac{4}{2(2)} = -1$ The axis of symmetry is $x = -1$.

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Example 3: Finding the Axis of Symmetry by Using the Formula

Find the axis of symmetry of the graph of $y = -3x^2 + 10x + 9$.

Step 1. Find the values of a and b.

$y = -3x^2 + 10x + 9$

$a = -3, b = 10$

Step 2. Use the formula.

$$x = -\frac{b}{2a}$$

$$x = -\frac{10}{2(-3)} = \frac{5}{3}$$

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Once you have found the axis of symmetry, you can use it to identify the vertex.

Finding the Vertex of a Parabola	
Step 1	To find the *x*-coordinate of the vertex, find the axis of symmetry by using zeros or the formula.
Step 2	To find the corresponding *y*-coordinate, substitute the *x*-coordinate of the vertex into the function.
Step 3	Write the vertex as an ordered pair.

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Example 4B: Finding the Vertex of a Parabola

Find the vertex.

$y = -3x^2 + 6x - 7$

Step 1 Find the x-coordinate of the vertex.

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Example 4B Continued

Find the vertex.

$y = -3x^2 + 6x - 7$

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