Identifying Quadratic Functions

The quadratic function $y = x^2$ does not have constant first differences. It has constant second differences. This is true for all quadratic functions.

A. Tell whether the function $y + 6x^2 = 5x$ is quadratic. Explain.

Try writing the function in the form $y = ax^2 + bx + c$ by solving for y.

$y + 6x^2 = 5x$

$-6x^2 \quad -6x^2$ — Subtract $6x^2$ from each side.

$y = -6x^2 + 5x$ — Write in $ax^2 + bx + c$ form.

So, $a =$ -6, $b = 5$, and $c =$ 0.

Is the function quadratic? Yes How do you know?

When in standard there is x^2

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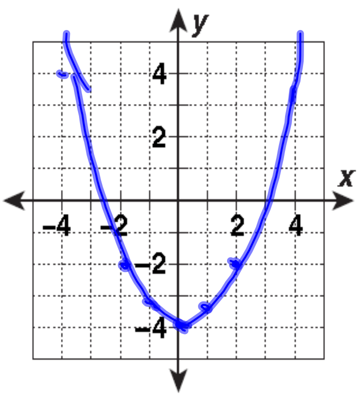
B. Graph the function $y = \frac{1}{2}x^2 - 4$ and give the domain and range.

Make a table of values. Choose values of x and use them to find values of y. Graph the points and connect with a smooth curve.

x	$y = \frac{1}{2}x^2 - 4$
-4	4
-2	-2
0	-4
2	-2
4	4



Is the value of a positive or negative? +

Therefore, the graph opens upward

The vertex is located at (0, -4).

Is the vertex a maximum or a minimum? MIN

The domain is all Real numbers. All the y-values of the function are greater than or equal to -4. So the range is $y \geq -4$.

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9A *9-2 Characteristics of Quadratic Functions*

The x-coordinate of the vertex of a parabola can be found by $x = -\frac{\boxed{b}}{2a}$.

The height above water of a curved arch support for a bridge can be modeled by $y = -0.004x^2 + 0.68x + 0.6$, where x is the distance in feet from where the arch support enters the water. How tall is the arched bridge?

Understand the Problem

1. What are you being asked to find? Vertex for MAX
2. What formula are you given? $\frac{-b}{2a}$ axis of sym.

Make a Plan

3. The Vertex/Max represents the highest point of a parabola.
4. The formula for the vertex of a parabola is $x = -\frac{\boxed{b}}{2a}$.

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Solve

5. Given the equation $y = -0.004x^2 + 0.68x + 0.6$,

   $a =$ -.004 and $b =$ .68.

6. Substitute the values for a and b into the vertex formula.

   $x = -\frac{\boxed{.68}}{2a}$ -.68/(2(-.004))

   $x = -\frac{\boxed{.68}}{2(-0.004)} =$ 85 Axis of sym.

7. Find the corresponding y-coordinate. $y = -0.004x^2 + 0.68x + 0.6$

   $= -0.004(85)^2 + 0.68(85) + 0.6$

   $=$ 29.5

8. The height of the bridge is 29.5 feet.

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Look Back

9. Graph the function $y = -0.004x^2 + 0.68x + 0.6$ on a graphing calculator. Viewing window: x-values -5 to 200 by 25, y-values -10 to 50 by 10.

10. Use the Calc feature on your calculator and determine the maximum point on the parabola. 29.5

11. Do your heights in Exercise 8 and Exercise 10 match? Si

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Finding Zeros of Quadratic Functions From Graphs

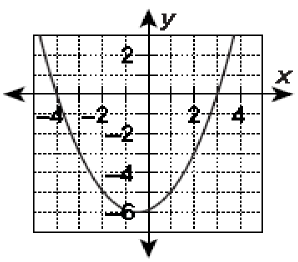
Find the zeros of the quadratic function from its graph. Then find its axis of symmetry.

The zero of a function is an x-value that makes the function equal to zero.
The zero of a function is the same as an x-intercept. solution

A quadratic function may have one, 2, or no zeros.

The axis of symmetry always passes through the Vertex of the parabola.

A.



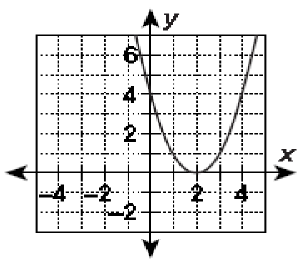
Where does the graph cross the x-axis?

-4 and 3

To determine the axis of symmetry, find the average of the zeros.

$\frac{-4 + \boxed{3}}{2} = -\frac{\boxed{1}}{2}$ $x =$ -1/2

B.



Where does the graph cross the x-axis?

2

In this case, the x-coordinate is the axis of symmetry.

$x =$ 2

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Finding the Vertex of a Parabola

Find the vertex of the parabola $y = -2x^2 + 4x - 3$.

Step 1 Find the x-coordinate using the formula $x = -\frac{b}{2a}$.

What does a equal? -2 What does b equal? 4

$$x = \frac{-b}{2a} = -\frac{4}{2(\boxed{-2})} = -\frac{4}{\boxed{-4}} = 1$$

Step 2 Find the corresponding y-coordinate.

$y = -2x^2 + 4x - 3$

$y = -2(1)^2 + 4(1) - 3$

$y = -2 + 4 - 3 = -1$

Step 3 Write the coordinates as an ordered pair. The vertex is $(1, -1)$.

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Graphing a Quadratic Function

Graph $y = 2x^2 + 4x + 1$.

Step 1 Find the axis of symmetry.

$x = -\frac{b}{2a}$ What does a equal? 2 What does b equal? 4

$x = -\frac{4}{2(\boxed{2})} = -\frac{4}{\boxed{4}} = -1$ Substitute known values and solve for x.

Step 2 Find the vertex. Substitute the x-coordinate into the equation.

$y = 2x^2 + 4x + 1$

$y = 2(-1)^2 + 2(-1) + 1$

$y = 2 - 4 + 1$ 4

$y = -1$ The vertex is $(-1, -1)$.

Step 3 Find the y-intercept.

$y = 2x^2 + 4x + 1$ Identify c. 1

The y-intercept is 1; the graph passes through (0, ____).

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Step 4 Find two more points on the same side of the axis of symmetry as the point containing the y-intercept. Use -3 and -2.

Let $x = -3$

$y = 2x^2 + 4x + 1$

$y = 2(\underline{-3})^2 + 4(\underline{-3}) + 1$

$y = 2(\underline{9}) - \underline{12} + 1$

$y = \underline{7}$

The point is $(-3, \underline{7})$.

Let $x = -2$

$y = 2x^2 + 4x + 1$

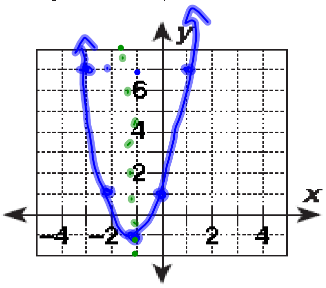
$y = 2(\underline{-2})^2 + 4(\underline{-2}) + 1$

$y = 2(\underline{4}) - \underline{8} + 1$

$y = \underline{1}$

The point is $(-2, \underline{1})$.

Step 5 Graph the axis of symmetry, vertex, y-intercept, and the two other points.



Step 6 Now, reflect the points across the axis of symmetry to graph points on the other side of the parabola. Connect the points with a smooth curve.

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Given a quadratic equation, you can write and graph the related function to determine the zeros of the function.

The height of a water rocket is launched upward with an initial velocity of 48 feet per second. Its height h, from the ground can be approximated by $h = -16t^2 + 48t$, where t is the time in seconds. Find the time it takes for the rocket to reach the ground after it is launched.

Understand the Problem

1. What are you being asked to determine?

   How long was rocket in air

2. What equation approximates the height? $-16t^2 + 48t$

Make a Plan

3. Write the related function and graph the function.

   The related function: $h = -16t^2 + 48t$

   $0 = -16t^2 + 48t$

   $\underline{0} = -16t^2 + 48t$

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Solve

4. Graph the function from Exercise 3. Use a graphing calculator.
5. Use the TRACE key to estimate the zeros.

   The zeros appear to be 0 and 3.
6. The rocket leaves the launch pad at 0 seconds and reaches the ground in 3 seconds.
7. The rocket is in the air for about 3 seconds.

Look Back

8. Substitute your value in Exercise 7 for t and see if the answer checks.

$0 = -16t^2 + 48t$

0	$-16(3)^2 + 48(3)$
0	$-16(3) + 144$
0	$-144 + 144$
0	0 ✓

Solving Quadratic Equation by Graphing

Solve each equation by graphing the related function.

A. $x^2 - 4$

Step 1 Write the related function. $y = x^2 - 4$

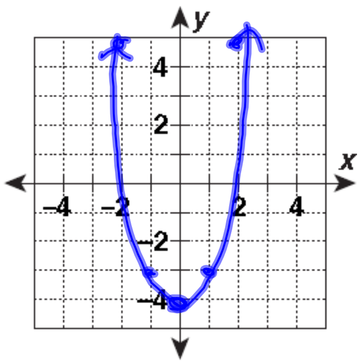
Step 2 Graph the function.

$\frac{0}{?}$

The axis of symmetry is $x = 0$.

The vertex is $(0, -4)$.

Two other points are $(-1, -3)$ and $(-3, 5)$.
Graph the points and reflect them across the axis of symmetry.



Step 3 Find the zeros. The zeros appear to be -2 and 2.

Check

$x^2 - 4$	$= 0$
$(-2)^2 - 4$	0
$4 - 4$	0
0	0

$x^2 - 4$	$= 0$
$(2)^2 - 4$	0
$4 - 4$	0
0	0

B. $x^2 + 3x - 10$

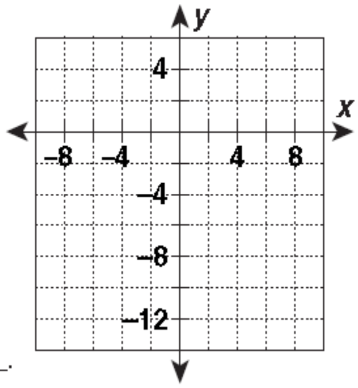
Step 1 Write the related function. $y = x^2 +$ 3x − 10

Step 2 Graph the function.

The axis of symmetry is $x =$ $-\frac{3}{2}$.

The vertex is $(-1.5,$ ________).

Two other points are $(-4,$ ______) and $(-3,$ ______). Graph the points and reflect them across the axis of symmetry.



Step 3 Find the zeros. The zeros appear to be ______ and ______.

Check

$x^2 + 3x - 10$	0
$(-5)^2 + 3($______$) - 10$	0
$25 -$ ______ $- 10$	0
0	0

$x^2 + 3x - 10$	0
$(2)^2 + 3($______$) - 10$	0
$4 +$ ______ $- 10$	0
0	0

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Using the Zero Product Property

If the product of two quantities equal zero, at least one of the quantities equals zero. If a quadratic equation is written in standard form, $ax^2 + bx + c = 0$, then to solve the equation, you may need to simplify / factor before using the Zero Product Property.

Use the Zero Product Property to solve each equation.

A. $(x + 7)(x + 3) = 0$

$x + 7 = 0$ or $x + 3 = 0$ — Use the Zero Property.

$x = -7$ or $x = -3$ — Solve each equation.

Check: $x = -7$

$(x + 7)(x + 3) = 0$

$(-7 + 7)(-7 + 3) = 0$

$(0)(-4) = 0$

$0 = 0$

Check: $x = -3$

$(x + 7)(x + 3) = 0$

$(-3 + 7)(-3 + 3) = 0$

$(4)(0) = 0$

$0 = 0$

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B. $x(x - 3) = 10$

$x^2 - \underline{3x} = 10$ Multiply.

$x^2 - 3x - 10 = \underline{0}$ Write the equation in Standard form

$(x - \underline{5})(x + \underline{2}) = 0$ Factor.

$x - 5 = 0$ or $x + \underline{2} = 0$ Use the Zero Product Property.

$x = \underline{5}$ or $x = \underline{-2}$ Solve each equation.

C. $(x + 5)(x - 8) = -22$

$x^2 - \underline{\quad} + 5x - \underline{\quad} = -22$ Multiply using the FOIL method.

$x^2 - \underline{\quad} - 40 = -22$ Combine like terms.

$x^2 - 3x - \underline{\quad} = 0$ Write the equation in standard form.

$(x - \underline{\quad})(x + \underline{\quad}) = 0$ Factor the trinomial.

$x - \underline{\quad} = 0$ or $x + \underline{\quad} = 0$ Use the Zero Product Property.

$x = \underline{6}$ or $x = \underline{-3}$ Solve each equation.

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Using Square Roots to Solve Quadratic Equations

Every positive real number has 2 square roots, one + and one −. When you take the square root of a positive real number and the sign of the square root is not indicated, you must find both the positive and negative square root. This is indicated by ± $\sqrt{\ }$. The square root of 0 is neither positive nor negative. (Only when b=0)

Solve using square roots. Round to the nearest hundredth if necessary.

A. $6x^2 = 216$

$\frac{6x^2}{\boxed{6}} = \frac{216}{\boxed{6}}$ Divide each side by 6.

$x^2 = \underline{36}$

$\sqrt{x^2} = \sqrt{36}$ Solve for x by taking the sq. root of both sides.

$x = \pm\underline{6}$ Use ± to show both square roots.

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B. $18 = x^2 - 31$

$18 = x^2 - 31$

$+31 \quad + \_\_\_$

$\_\_\_ = x^2$

$\sqrt{\_\_\_} = \sqrt{x^2}$

$x = \pm 7$

Add 31 to each side of the equation.

Solve for x by taking the square root of both sides.

Use $\pm$ to show both square roots.

C. $5x^2 + 6 = 34$

$5x^2 + 6 = 34$

$-\_\_\_ \quad -\_\_\_$

$5x^2 = \_\_\_$

$\frac{5x^2}{\square} = \frac{28}{\square}$

$x^2 = \frac{\square}{5}$

$\sqrt{x^2} = \sqrt{\frac{\square}{5}}$

$x = \pm 2.37$

Subtract 6 from each side of the equation.

Divide each side by 5.

Solve for x by taking the sq. root of each side.

Use $\pm$ to show both square roots. Round to the nearest hundredth.

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Using the Quadratic Formula

Solve $x^2 + 7x + 2 = 0$ using the Quadratic Formula.

What is the quadratic formula?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the equation $x^2 + 7x + 2 = 0$, $a = 1$ $b = 7$ and $c = 2$.

Substitute for a, b, and c in the quadratic formula.

$$x = \frac{-\boxed{7} \pm \sqrt{\boxed{7}^2 - 4(1)\boxed{2}}}{2(1)}$$

$$x = \frac{-\boxed{7} \pm \sqrt{\boxed{49} - 8}}{2(1)}$$ Simplify.

$$x = \frac{-\boxed{7} \pm \sqrt{\boxed{41}}}{2}$$

$$x = \frac{-7 + \sqrt{\boxed{41}}}{2} \text{ or } x = \frac{-7 - \sqrt{\boxed{41}}}{2}$$

$x \approx -.298$ $\quad$ $x \approx 6.702$

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Analyzing Quadratic Equations by Using the Discriminant

Find the type and number of solutions for the equation $3x^2 - 4x + 5 = 0$.

If $b^2 - 4ac$ is positive, there are 2 real solutions.

If $b^2 - 4ac = 0$, then there is 1 real solution.

If $b^2 - 4ac$ is negative, then there are NO real solutions.

For the equation $3x^2 - 4x + 5 = 0$, $a =$ 3, $b =$ -4 and $c =$ 5.

Substitute values for a, b and c into the discriminant formula $b^2 - 4ac$.

$$b^2 - 4ac = (-4)^2 - 4(3)(5)$$
$$= 16 - 60$$
$$= -44$$

The equation $3x^2 - 4x + 5 = 0$ has NO real solutions.

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