Definitions

A *rational function* is a function whose rule has a variable in the denominator.

$y = \frac{1}{x}$

An *excluded value* for a rational function is any value that makes the denominator equal to 0.

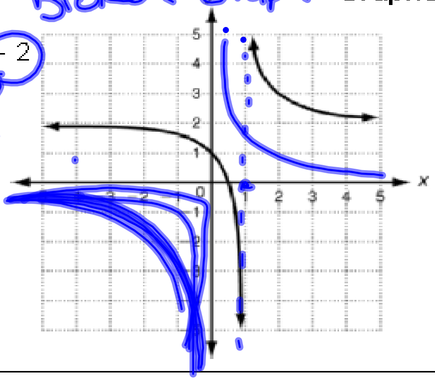
Domain

Graphs

Broken Graph

$y = \frac{1}{x-1} + 2$

EV: $x = 1$



Rational Function

Examples

$y = \frac{4}{x}$; $y = \frac{1}{x^2}$; $y = \frac{2}{x-5} + 3$

Non-Examples

$y = \frac{x}{6}$; $y = x + \frac{1}{2}$; $y = 3x^2$

Asymptotes

A rational function in the form $y = \frac{a}{x+b} + c$ has a *vertical asymptote* at the excluded value $x = -b$, and a *horizontal asymptote* at $y = c$.

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Complete the following.

1. Tell whether or not each of the following is a rational function.

a. $y = \frac{x}{3} + 2$ NO

b. $y = \frac{3}{x^2} - 1$ Yes

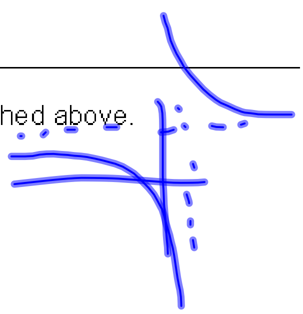
c. $y = \frac{1}{x+8}$ Yes

2. Explain why $y = x + \frac{1}{2}$ is not a rational function. No variable in denom.

3. Describe how to find the excluded value of the rational function $y = \frac{3}{x-5}$. Then find it. Set the denom = Ø

$x - 5 = 0$ $x = 5$

4. Identify the asymptotes for the rational function $y = \frac{1}{x-1} + 2$ graphed above.



vertical: = EV $x = 1$ $x - 1 = 0$

horizontal: = C $y = 2$

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The table below shows examples of the many ways that rational expressions can be simplified.

Use Properties of Exponents	Factor out Common Monomials	Factor Trinomials & Special Products	Use Opposite Binomials
$\frac{4m^5}{24m^2}$ $\frac{m^3}{6}$	$\frac{2b^2-18b}{b-9}$ $\frac{2b(b-9)}{b-9}$ $2b$	$\frac{x^2-9}{x^2+2x-15}$ $\frac{(x+3)(x-3)}{(x+5)(x-3)}$ $\frac{x+3}{x+5}$	$\frac{6t-t^2}{t^2-4t-12}$ $\frac{t(6-t)}{(t+2)(t-6)}$ $\frac{t(-1)(t-6)}{(t+2)(t-6)}$ $-\frac{t}{(t+2)}$

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Use the table to answer the following.

1. Look at the first column. Explain how $\frac{4m^5}{24m^2}$ was simplified to $\frac{m^3}{6}$. Cancel 4, + m^2
2. Look at the second column. What expression was divided out of the numerator and denominator? b-9
3. Look at the third column. What type of special product was in the numerator? Diff. of Sq.
4. Look at the fourth column. What happened in the numerator? factored -t

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Simplify each rational expression.

5. $\dfrac{3t^4}{9t^6}$

$\dfrac{1}{3t^2}$

6. $\dfrac{n^4+3n^3}{2n^2+6n}$

$\dfrac{n^3(n+3)}{2n(n+3)} = \dfrac{n^3}{2n}$

7. $\dfrac{4c+4}{2c^2-5c-7}$

$\dfrac{4(c+1)}{(2c-7)(c+1)} = \dfrac{4}{(2c-7)}$

	$2c$	-7
c	$2c^2$	$-7c$
1	$2c$	-7

-14

$1 \quad 14$

$2 \quad -7$

8. $\dfrac{18-2x}{x^2-81}$

$-2x+18$

$\dfrac{-2(x-9)}{(x+9)(x-9)} = \dfrac{-2}{x+9}$

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FRACTIONS

Top · Top
bottom · bottom

Rule 1: $\dfrac{a}{b}\cdot\dfrac{c}{d}=\dfrac{ac}{bd}$

RATIONAL EXPRESSIONS

$\dfrac{8}{9}\cdot\dfrac{3}{5}=\dfrac{8\cdot3}{9\cdot5}=\cdots$

>1 term
use ( )

$\dfrac{m+2}{4}\cdot\dfrac{8}{3m+6}=\dfrac{8(m+2)}{4(3m+6)}=\cdots$

Rule 2: $\dfrac{a}{b}\cdot c=\dfrac{a}{b}\cdot\dfrac{c}{1}$ and $\dfrac{a}{b}\div c=\dfrac{a}{b}\div\dfrac{c}{1}$

$\dfrac{3}{4}\cdot 16=\dfrac{3}{4}\cdot\dfrac{16}{1}=\cdots$

$\dfrac{6}{2x+10}\cdot(x^2+3x-10)=\dfrac{6}{(2x+10)}\cdot\dfrac{(x^2+3x-10)}{1}=\cdots$

KCF

Rule 3: $\dfrac{a}{b}\div\dfrac{c}{d}=\dfrac{a}{b}\cdot\dfrac{d}{c}$

$\dfrac{1}{12}\div\dfrac{2}{3}=\dfrac{1}{12}\cdot\dfrac{3}{2}=\cdots$

$\dfrac{3ab^4}{a^3}\div\dfrac{3ab}{ab^2}=\dfrac{3ab^4}{a^3}\cdot\dfrac{ab^2}{3ab}=\cdots$

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Finish multiplying and dividing the rational expressions given above so that each is simplified completely. ?

1. Rule 1

$\frac{8(m+2)}{4(3m+6)}$

2. Rule 2

$\frac{6(x^2+3x-10)}{2x+10}$

3. Rule 3

$\frac{3a^2b^6}{3a^4b}$

Multiply or divide. Simplify your answer.

4. $\frac{3}{t+2}\cdot(t^2-4)$ diff □

$\frac{3(t+2)(t-2)}{t+2}$

5. $\frac{m+5}{4}\div(3m+15)$ $\cdot\frac{1}{(3m+15)}$

$\frac{m+5}{4(3m+15)}$

6. $\frac{x^2y^3}{8y}\cdot\frac{10x^4}{xy^2}$

$\frac{10x^6y^3}{8xy^3}$

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Add $\frac{3}{9x}+\frac{x}{6x^3}$.

Only + or − w/ common denominato

$9x = 3\cdot 3\cdot x$

$6x^3 = 2\cdot 3\cdot x\cdot x\cdot x$

LCD: $2\cdot 3\cdot 3\cdot x\cdot x\cdot x = 18x^3$

1 Identify the least common denominator, LCD.

$\frac{3}{9x}\left(\frac{2x^2}{2x^2}\right)+\frac{x}{6x^3}\left(\frac{3}{3}\right)$

2 Multiply expressions by appropriate form of 1.

$\frac{6x^2}{18x^3}+\frac{3x}{18x^3}$

3 Write each expression with the new LCD.

$\frac{6x^2+3x}{18x^3}$

4 Add/subtract numerators. Keep denominator.

$\frac{3x(2x+1)}{18x^3}$

5 Factor the numerator and/or denominator.

$\frac{2x+1}{6x^2}$

6 Simplify as needed.

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Answer each question about the procedure shown above.

1. If the two expressions have the same denominator, which step can you start with? ______________

2. Why was $\left(\frac{2x^2}{2x^2}\right)$ the "appropriate form of 1" for the first expression?

____

3. How would the answer to this problem change if it were subtraction instead of addition?

____

Add or subtract.

4. $\frac{x+5}{x^2-9}-\frac{2}{x^2-9}$

5. $\frac{1}{4x}+\frac{4x}{6x^2}$

__________________________________ __________________________________

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